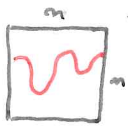
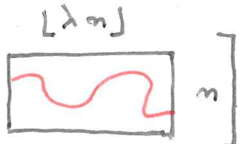
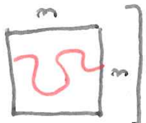
# SHEET 8.

**Exercise 1** : Consider percolation on $\mathbb{Z}^2$.

Let $\lambda > 0$. Prove that there exists $h_\lambda : [0,1] \longrightarrow [0,1]$ continuous increasing (strictly) s.t. $h_\lambda(0)=0$, $h_\lambda(1)=1$ and for every $p \in [0,1]$ $\forall n \geq 1/\lambda$.

$h_\lambda^{-1}\Big(\mathbb{P}_p\Big[$  $\Big]\Big) \geq \mathbb{P}_p\Big[$  $\Big] \geq h_\lambda\Big(\mathbb{P}_p\Big[$  $\Big]\Big)$

**Exercise 2** $G = (\mathbb{Z}^d, E)$ let $k \geq 1$.

We consider a random variable $X = (X(e))_{e \in E} \in \{0,1\}^E$.

We assume that $X$ is a $k$-dependent percolation. That is.

$$\forall A, B \subset E \quad \text{s.t.} \quad \forall e \in A \ \forall f \in B \quad \underset{\text{graph distance}}{d(e,f)} > k$$

$(X(e))_{e \in A}$ and $(X(e))_{e \in B}$ are independent.

1. Prove that there exists $\delta_k > 0$ small enough s.t.

$$\big(\forall e \quad \mathbb{P}(X(e)=1) \leq \delta_k\big) \Rightarrow \big(\exists c>0 \ \forall n \geq 1 \ \mathbb{P}[X \in (0 \longleftrightarrow \partial\Lambda_n)] \leq e^{-cn}\big)$$

2. Prove that

$$\big(\forall e \quad \mathbb{P}(X(e)=1) \geq 1-\delta_k\big) \Rightarrow \big(\mathbb{P}[X \in (0 \longleftrightarrow \infty)] > 0\big)$$

Hint: prove it first for $\mathbb{Z}^2$.