

SHEET 9

P_p percolation on \mathbb{Z}^2 .

Exercise 1

Let $p > \frac{1}{2}$. Prove that $P_p \left[\text{rectangle } \begin{matrix} 3n \\ n \end{matrix} \right] \xrightarrow{n \rightarrow \infty} 1$.

(One can give two different proofs).

Exercise 2

Let $p = \frac{1}{2}$. Define $\pi(m, n) = P_{\frac{1}{2}} [\Lambda_m \leftrightarrow \partial \Lambda_n]$ for $m \leq n$.

Prove that there exists a constant $c > 0$ s.t. $\forall n_3 \geq n_2 \geq 2n_1$,

$$c \pi(n_1, n_2) \pi(n_2, n_3) \leq \pi(n_1, n_3) \leq \pi(n_1, n_2) \pi(n_2, n_3)$$

Exercise 3 (difficult)

Prove that there exist $c_1, c_2 > 0$ s.t. $\forall n \geq 1$

$$c_1 \cdot \frac{1}{n^2} \leq P_{\frac{1}{2}} \left[\text{rectangle } \begin{matrix} 2n \\ n \end{matrix} \right] \leq c_2 \cdot \frac{1}{n^2}$$