I. Green's formula.

1. If $\gamma \subset \mathbb{R}^2$ is the oriented boundary (positively oriented) of $\Omega = \mathbb{R}^2 \cap \{(x,y) : 0 \le x \le 1,\ x^2 \le y \le \sqrt{x}\}$, and $F(x,y) = ((2xy - x^2), (x + y^2))$. Compute
$$\int_\gamma F \cdot d\vec{s}.$$
2. Let $0 < a, b < \infty$ and $E_{a,b} = \mathbb{R}^2 \cap \left\{(x,y) :\ x \ge 0,\ y \ge 0,\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\}$ be the quarter of ellipse. Compute
$$\int_{E_{a,b}} (2x^3 - y)dxdy.$$

II. Area computation.

Let $a > 0$ and Ω_a be the compact domain delimited by the arcs $y = 0$, and

$$(x(t), y(t)) = (a(t - \sin(t)), a(1 - \cos(t))), \quad t \in [0, 2\pi].$$

Compute the area of Ω_a.

III. Two methods.

Let $K = \mathbb{R}^2 \cap \{(x,y) : x \ge 0,\ y \ge 0 \text{ and } x^2 + y^2 \le 1\}$, and γ its oriented boundary, and $F : \mathbb{R}^2 \to \mathbb{R}^2$ the vector field such that for all $x, y \in \mathbb{R}$, we have $F(x,y) = (xy^2, 2xy)$.

Compute $\displaystyle\int_\gamma F \cdot d\vec{s}$:

1. By using a parametrisation of γ.
2. By using the Green's formula.