I. Conservative vector-fields

Which of the following vector-fields v are conservative?

1. $v(x,y) = \begin{pmatrix} x-y \\ x-y \end{pmatrix}$
2. $v(x,y) = \begin{pmatrix} x^2-y \\ x^3+2xy \end{pmatrix}$
3. $v(x,y) = \begin{pmatrix} x^3+2xy \\ x^2-y \end{pmatrix}$
4. $v(x,y) = \begin{pmatrix} x^3-xy^2 \\ x^2y-y^5 \end{pmatrix}$

For those that are conservative, compute a potential.

II. A counter-example Let $V : \mathbb{R}^2 \backslash \{0\} \to \mathbb{R}^2$ be defined by

$$V(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right)$$

1. Show that V satisfies the necessary condition of Proposition 4.1.13 of the script to be conservative.
2. Compute the line integral of V around the curve $\gamma(t) = (\sin(t), -\cos(t))$, $0 \le t \le 2\pi$.
3. Is V conservative?

III. Pathintegrals Compute the line integrals of the vector fields v along the indicated curves, choosing a simple parameterization when none is specified:

1. $v(x,y) = \begin{pmatrix} x^2-2xy \\ y^2-2xy \end{pmatrix}$, from $(-1,1)$ to $(1,1)$ along the path $y = x^2$.
2. $v(x,y) = \begin{pmatrix} x^2+y^2 \\ x^2-y^2 \end{pmatrix}$, from $(0,0)$ to $(2,0)$ along the path $y = 1 - |1-x|$.
3. $v(x,y,z) = \begin{pmatrix} x \\ y \\ xz-y \end{pmatrix}$, along the path $\gamma(t) = (t^2, 2t, 4t^3)$, $t \in [0,1]$.
4. $v(x,y) = \begin{pmatrix} 2a-y \\ x \end{pmatrix}$, along the path $\gamma(t) = (a(t-\sin(t)), a(1-\cos(t)))$, $t \in [0, 2\pi]$, where $a \in \mathbb{R}$ is a constant.
5. $v(x,y) = \left(\frac{x}{x^2+y^2+1}, \frac{y}{x^2+y^2+1} \right)$, along the circle $x^2+y^2-2x = 1$.
6. $v(x,y,z) = \begin{pmatrix} 2xy^2z \\ 2x^2yz \\ x^2y^2-2z \end{pmatrix}$, along the path $\gamma(t) = \left(\cos(t), \frac{\sqrt{3}}{2}\sin(t), \frac{1}{2}\sin(t) \right)$, $0 \le t \le 2\pi$.