Analysis III (BAUG) **Assignment 12**

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The first 4 questions of this exercise sheet review the topics connected to beams (static and dynamics). You should try to solve at least one point for each of them.

Question 5 is Exercise 4.1 of Lecture 10. Giving a full correct solution may be not easy, but it is a good exercise to think at least a little bit about it.

Question 1:

For each of the following pairs of functions f, g, find the solution to the following IBVP:

$$\begin{aligned} \text{PDE}: &\quad u_{tt}(x,t) = -u_{xxxx}(x,t) && \text{for } 0 < x < 1 \text{ and } t > 0 \\ \text{BC}: &\quad u(0,t) = 0,\ \ u_{xx}(0,t) = 0,\ \ u(1,t) = 0,\ \ u_{xx}(1,t) = 0 && \text{for } t \geq 0 \\ \text{IC}: &\quad u(x,0) = f(x),\ \ u_t(x,0) = g(x) && \text{for } 0 \leq x \leq 1 \end{aligned}$$

(i) $f(x) = g(x) = \sin(\pi x)$ for $x \in [0,1]$.

(ii) $f(x) = 1 - x^2$ and $g(x) = 0$ for $x \in [0,1]$.

Question 2:

Suppose we have a beam of length 10 (parametrized by $0 \leq x \leq 10$) embedded in the wall at one end, free at the other, and suppose $EI = 1$.

For each of the scenarios below, find the deflection curve by first writing up the corresponding ODE with the corresponding boundary conditions (see hints for answer), and then solving the obtained boundary value problem using the Laplace transform.

(i) We apply force $F = 1$ downwards at the point $x = 5$.

(ii) The force we apply is described by the function

$$f(x) = \begin{cases} 0 & \text{for } x < 4 \\ 8 - 2x & \text{for } 4 \leq x. \end{cases}$$

Question 3:

Suppose we have a beam of length 3 (parametrized by $0 \leq x \leq 3$) simply supported at both ends, and suppose $EI = 4$

For each of the scenarios below, find the deflection curve by first writing up the corresponding ODE with the corresponding boundary conditions (see hints for answer), and then solving the obtained boundary value problem using the Laplace transform.

(i) We simultaneously apply force $F = 1$ downwards at both of the points $x = 1$ and $x = 2$.

(ii) The force we apply is described by the function

$$f(x) = \begin{cases} 1 & \text{for } x < 2 \\ 0 & \text{for } 2 \leq x. \end{cases}$$

Question 4:

Suppose we have a water tower (beam column) of length 10 (parametrized by $0 \leq x \leq 10$) and load $W = 10$ and $EI = 10$. Suppose that the lateral forces are described by the following function:

$$f(x) = \begin{cases} 0 & \text{for } x < 5 \\ 20 & \text{for } 5 \leq x. \end{cases}$$

What is the corresponding ODE for the deflection curve. Solve the ODE to find the deflection curve.

You can use the following formulas in your solutions:

$$\mathcal{L}^{-1}(\frac{1}{s(s^2+1)}) = 1 - \cos(x)$$

$$\mathcal{L}^{-1}(\frac{1}{s^2(s^2+1)}) = x - \sin(x)$$

$$\mathcal{L}^{-1}(\frac{1}{s^3(s^2+1)}) = -1 + \frac{x^2}{2} + \cos(x)$$

Question 5:

The goal of this exercise is to show that for any $a > 0$, the Dirac delta function $\delta(t-a)$ behave as the "derivative" of the Heaviside step function $u(t-a)$. Recall that the delta function is defined to be such that for each integrable function f, the following holds:

$$\int_0^\infty f(t)\delta(t-a)dt = f(a).$$

Show that the "derivative" of the Heaviside step function behaves as δ.