Analysis III (BAUG) **Assignment 9**

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The goal of this exercise sheet is to familiarize with the Laplace transform. There are two types of exercises, grouped in Questions 1 and 2 respectively. Given the importance of the topic, there is no hard question this time: focus on practising the Laplace transform.

Question 1

Find the inverse f of each of the following Laplace transforms F.

(i) $F(s) = \frac{-2}{s+3}$

- ☐ $f(t) = -8$
- ☐ $f(t) = -2e^{-3t}$
- ☐ $f(t) = -2e^{3t}$
- ☐ $f(t) = \frac{-2e^{-3t}}{t}$
- ☐ $f(t) = -6e^{3t}$

(ii) $F(s) = \frac{3s+13}{s^2+4s+3}$

- ☐ $f(t) = 5e^{t} + 2e^{3t}$
- ☐ $f(t) = 5e^{-t} + 2e^{-3t}$
- ☐ $f(t) = 5e^{t} - 2e^{3t}$
- ☐ $f(t) = 5e^{-t} - 2e^{-3t}$

(iii) $F(s) = \frac{s^2+1}{s^2(s+1)}$

- ☐ $f(t) = 2e^{t} + t^2 - t$
- ☐ $f(t) = 2e^{t} + t - 1$
- ☐ $f(t) = 2e^{-t} + t^2 - t$
- ☐ $f(t) = 2e^{-t} + t - 1$

(iv) $F(s) = \frac{s+1}{s^2+4}$

- ☐ $f(t) = \cos(2t) + \frac{1}{2}\sin(2t)$
- ☐ $f(t) = \cos(2t) + \sin(2t)$
- ☐ $f(t) = e^{-t}\cos(2t)$
- ☐ $f(t) = e^{t}\cos(2t)$

(v) $F(s) = \frac{s^3+s+1}{(s^2+1)(s^2+4)}$

- ☐ $f(t) = \frac{1}{3}\sin(t) + \cos(2t) + \frac{1}{6}\sin(2t)$
- ☐ $f(t) = \frac{1}{3}\sin(t) + \cos(2t) - \frac{1}{6}\sin(2t)$
- ☐ $f(t) = -\frac{1}{3}\sin(t) + \cos(2t) + \frac{1}{6}\sin(2t)$
- ☐ $f(t) = -\frac{1}{3}\sin(t) - \cos(2t) + \frac{1}{6}\sin(2t)$

(vi) $F(s) = \frac{s+1}{(s+1)^2+4}$

- ☐ $f(t) = \cos(2t) + \frac{1}{2}\sin(2t)$
- ☐ $f(t) = \cos(2t) + \sin(2t)$
- ☐ $f(t) = e^{-t}\cos(2t)$
- ☐ $f(t) = e^{t}\cos(2t)$

(vii) $F(s) = \frac{2s}{s^2+2s+5}$

- ☐ $f(t) = 2e^{-t}\cos(2t) - e^{-t}\sin(2t)$
- ☐ $f(t) = 2e^{-t}\cos(2t) + e^{-t}\sin(2t)$
- ☐ $f(t) = 2e^{t}\cos(2t) - e^{t}\sin(2t)$
- ☐ $f(t) = 2e^{t}\cos(2t) + e^{t}\sin(2t)$

Question 2

Use the Laplace transform to solve the following initial value problems.

(i) $x''(t) + 4x'(t) + 3x(t) = 0, \qquad x(0) = 3, \qquad x'(0) = 1$

(ii) $x'(t) + x(t) = t, \qquad x(0) = 1$

(iii) $x''(t) + 4x(t) = \sin(t), \qquad x(0) = 1, \qquad x'(0) = 0$

(iv) $x''(t) + 2x'(t) + 5x(t) = 0, \qquad x(0) = 2, \qquad x'(0) = -4$