Analysis III (BAUG) **Assignment 1**

Prof. Dr. Alessandro Sisto — Due 27 of September 2018

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Note: No hard exercise for this week.

1. Verify that the each of the following functions satisfy the PDE "$u_{rr} + u_{tt} = 0$".
   - $u(r,t) = e^{2r}\cos 2(t)$
   - $u(r,t) = 3r^2t - t^3$
   - $u(r,t) = \sin(r)\cosh(t)$.
   - $u(r,t) = \log(r^2+t^2)$ for $r^2+t^2 \neq 0$
   - $u(r,t) = e^r\cos(t) + 3r^2t - t^3 + \sin(r)\cosh(t)$

2. Verify that the each of the following functions satisfy the PDE "$u_{\theta\theta} - c^2u_{rr} = 0$", where c is a real constant.
   - $u(r,\theta) = \sin(r - c\theta)$
   - $u(r,\theta) = \log(r + c\theta)$ for $r + c\theta > 0$
   - $u(r,\theta) = \cos(ar)\sin(ca\theta)$ for any real constant a
   - $u(r,\theta) = e^{r+c\theta} + e^{r-c\theta}$

3. Verify that the each of the following functions satisfy the PDE "$u_t - ku_{xx} = 0$", where k is a real constant.
   - $u(x,t) = x^2 + 2kt$
   - $u(x,t) = e^{-kt}\sin(x)$
   - $u(x,t) = e^{kt}\cosh(x)$
   - $u(x,t) = e^{-a^2kt}\cos(ax)$ for any real constant a

4. Classify each of the following 2nd order PDEs according to their homogeneity (homogeneous or nonhomogeneous), linearity (linear or non linear), coefficients (constant or non constant) and, when the PDE is linear, type (parabolic, hyperbolic or elliptic).

   Note: An equation may have different types in different regions of the domain of the function.

   (a) $(1+y^2)u_{xx} + e^{-\frac{x^2}{2}}u_{yy} - xu_x + (3-x^2)u_y = 0$ [Past exam question]

   (b) $u_{xx} + 4xy + e^y u_y = (x+y)^2 u_x$ [Past exam question]

   (c) $u_{r\theta} - 6u_{rr} + uu_{\theta\theta} - e^r u_r = 4x$

   (d) $6u_{tt} + 12u_{rr} - 24u_r + 20u_t = 42$

   (e) $u_{xx} + \sin(x)u_x + e^{\pi y}u_y - 10xyu_{yy} = 0$

5. **(Calculus review)** Compute the following integrals.

(a) $\int_0^{\pi} \cos(nx)\, dx$, for n a positive integer

(b) $\int_{-\pi}^{\pi} x \sin(nx)\, dx$, for n a positive integer. [Past exam question]

(c) $\int_0^{2\pi} \cos(2x)\cos(x)\, dx$

(d) $\int_0^{2\pi} e^{-x} \sin(nx)\, dx$, for n an integer

(e) $\int_0^{2\pi} \sin(3x)^2\, dx$