D-MAVT
D-MATL

Prof. A. Iozzi
ETH Zürich
Autumn 2018

Analysis III

Serie 9

What's needed: Separation of variables, Fourier series solution of the 1-dimensional wave equation.

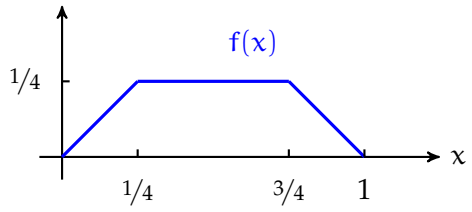
1. For $0 < k < 1$, find (via Fourier series) the solution $u = u(x,t)$ of the 1-dimensional wave equation with the following boundary and initial conditions:

$$\begin{cases} u_{tt} = u_{xx}, & \\ u(0,t) = 0 = u(1,t), & t \geqslant 0 \\ u(x,0) = kx(1-x^2), & 0 \leqslant x \leqslant 1 \\ u_t(x,0) = 0, & 0 \leqslant x \leqslant 1 \end{cases}$$

2. Find (via Fourier series) the solution $u = u(x,t)$ of the 1-dimensional wave equation with the following boundary and initial conditions:

$$\begin{cases} u_{tt} = u_{xx}, & \\ u(0,t) = 0 = u(1,t), & t \geqslant 0 \\ u(x,0) = f(x), & 0 \leqslant x \leqslant 1 \\ u_t(x,0) = 0, & 0 \leqslant x \leqslant 1 \end{cases}$$

where $f(x)$ is the following function



Please turn!

3. Find (via Fourier series) the solution $u = u(x,t)$ of the 1-dimensional wave equation with the following boundary and initial conditions:

$$\begin{cases} u_{tt} = u_{xx}, & \\ u(0,t) = 0 = u(\pi,t), & t \geqslant 0 \\ u(x,0) = 0, & 0 \leqslant x \leqslant \pi \\ u_t(x,0) = g(x), & 0 \leqslant x \leqslant \pi \end{cases}$$

where

$$g(x) = \begin{cases} \frac{x}{100}, & 0 \leqslant x \leqslant \frac{\pi}{2} \\ \frac{\pi - x}{100}, & \frac{\pi}{2} \leqslant x \leqslant \pi. \end{cases}$$

4. Find all possible solutions of the following PDEs of the form $u(x,t) = F(x)G(t)$ (separation of variables):

a) $xu_x + u_t = 0$

b) $u_x + u_t + xu = 0$

c) $t^3 u_x + \cos(x)u - 2u_{xt} = 0$

Due by: Thursday 22 / Friday 23 November 2018.