Referring to the cubic spline computation:

We have 2 pieces of information from the interpolating conditions:

① $\quad s_j(t_{j-1}) = \tilde{a}_j = y_{j-1}$

$\rightarrow$ this determines $\tilde{a}_j$

② We also have $s_j(t_j) = y_j$

and also $s_j(t_j) = \tilde{a}_j + \tilde{b}_j (t_j - t_{j-1}) + \tilde{c}_j (t_j - t_{j-1})^2$

$$+ \tilde{d}_j (t_j - t_{j-1})^3 \overset{(*)}{=} y_j$$

We already have checked $\tilde{c}_j = \dfrac{\tilde{\sigma}_{j-1}}{2}$,

$$\tilde{d}_j = \frac{\tilde{\sigma}_j - \tilde{\sigma}_{j-1}}{6 h_j} \quad \text{and}$$

$$\tilde{a}_j = y_{j-1}$$

Plug this into $(*)$:

$$\Rightarrow \quad y_{j-1} + \tilde{b}_j h_j + \frac{\tilde{\sigma}_{j-1}}{2} h_j^2 + \frac{\tilde{\sigma}_j - \tilde{\sigma}_{j-1}}{6 \cancel{h_j}} h_j^{\cancel{3}\,2} = y_j$$

$$\Rightarrow \quad \tilde{b}_j h_j = y_j - y_{j-1} - h_j^2 \left(\frac{2\tilde{\sigma}_{j-1}}{6} + \frac{\tilde{\sigma}_j}{6} \right)$$

$$\Rightarrow \quad \tilde{b}_j = \frac{y_j - y_{j-1}}{h_j} - \frac{h_j \left(2\tilde{\sigma}_{j-1} + \tilde{\sigma}_j \right)}{6}$$

$\uparrow$

Equation for $\tilde{b}_j$ verified.

<u>Next</u>: We use the matching of the 1st derivatives:

$$\tilde{b}_j + 2\tilde{c}_j h_j + 3\tilde{d}_j h_j^2 = \tilde{b}_{j+1}$$

to formulate an LSE for the $\tilde{\sigma}_j$'s.

We write

$$\frac{y_j - y_{j-1}}{h_j} - \frac{h_j \left(2\tilde{\sigma}_{j-1} + \tilde{\sigma}_j \right)}{6} + \tilde{\sigma}_{j-1} h_j + \frac{\tilde{\sigma}_j - \tilde{\sigma}_{j-1}}{2} h_j$$

$$= \frac{y_{j+1} - y_j}{h_{j+1}} - \frac{h_{j+1} \left(2\tilde{\sigma}_j + \tilde{\sigma}_{j+1} \right)}{6}$$

$$\Rightarrow \sigma_{j-1}\left(-\frac{h_j}{3} + h_j - \frac{h_j}{2}\right) + \sigma_j\left(-\frac{h_j}{6} + \frac{h_j}{2} + \frac{h_{j+1}}{3}\right)$$

$$+ \sigma_{j+1}\frac{h_{j+1}}{6} = \underbrace{\frac{y_{j+1} - y_j}{h_{j+1}} - \frac{y_j - y_{j-1}}{6}}_{=: r_j}$$

Then we obtain the LSE from the lecture.