Exercise Sheet 2

Exercise 1

Let G be a locally compact Hausdorff group. Show that

1. The modular function $\Delta_G : G \to \mathbb{R}_{>0}$ is continuous.
2. $\int_G f(x^{-1})\Delta_G(x^{-1})d\mu(x) = \int_G f(x)d\mu(x)$.

Exercise 2

Show that the modular function associated to the group

$$P = \{\begin{pmatrix} x & y \\ 0 & x^{-1} \end{pmatrix} | x > 0, y \in \mathbb{R}\}$$

is given by

$$\Delta_P \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix} = a^{-2}.$$

Exercise 3

Let G be a topological group and let H be a subgroup of G. Show that the map

$$G \times G/H \to G/H, \;\; (g, xH) \mapsto gxH,$$

is continuous.

Exercise 4

Let G be a locally compact Hausdorff group which is separable. Assume that $G \times X \to X$ is a transitive continuous action on a locally compact Hausdorff space X. Show that the map

$$G/\mathrm{Stab}_G(x_0) \to X, \;\; g\mathrm{Stab}_G(x_0) \mapsto gx_0$$

is a homeomorphism.