Exercise Sheet 5

Exercise 1

Show that $\mathfrak{so}(6,\mathbb{C}) \cong \mathfrak{sl}(4,\mathbb{C})$. (*Hint:* If $\dim V = 4$ then $\dim \Lambda^2 V = 6$.)

Exercise 2

Show that $\mathfrak{so}(4,\mathbb{C}) \cong \mathfrak{sl}(2,\mathbb{C}) \oplus \mathfrak{sl}(2,\mathbb{C})$.

Exercise 3

Show that $\mathfrak{so}(3,\mathbb{C}) \cong \mathfrak{sl}(2,\mathbb{C})$.

Exercise 4

Show that $\mathfrak{sp}(2,\mathbb{C}) \cong \mathfrak{sl}(2,\mathbb{C})$ and $\mathfrak{sp}(4,\mathbb{C}) \cong \mathfrak{so}(5,\mathbb{C})$.

Exercise 5

Let M be a smooth manifold and let X, Y be two complete vector fields on M. Prove that the following are equivalent

- The vector fields commute at every point of M, that is $[X,Y] = 0$.
- If we denote by $\Phi^X : \mathbb{R} \times M \to M$ (resp. Φ^Y) the flow associated to the vector field X (resp. Y) it holds $\Phi^X_t \circ \Phi^Y_s = \Phi^Y_s \circ \Phi^X_t$ for every $t, s \in \mathbb{R}$.