

8 Exercise Sheet 8

Exercise 8.1. Find two probability measures $\mu, \nu \in \mathcal{P}(\mathbb{R}^n)$ such that there exists a map (or a plan) from μ to ν that is optimal for the linear cost but not for the quadratic cost.

Exercise 8.2. Find two probability measures $\mu, \nu \in \mathcal{P}(\mathbb{R}^n)$ such that there exists a map (or a plan) from μ to ν that is optimal for the quadratic cost but not for the linear cost.

Exercise 8.3. Let $\mu, \nu \in \mathcal{P}(\mathbb{R}^n)$ be a pair of probability measures. Show that if μ and ν are supported on a compact set, then

$$\lim_{p \rightarrow 1} W_p(\mu, \nu) = W_1(\mu, \nu).$$

Show a counterexample to the previous statement if we drop the assumption that μ and ν are supported on a compact set.

Hint:

For the counterexample, set $\mu = \delta_0$ and find a measure ν such that $W_p(\mu, \nu) = \infty$ if $p > 1$ and $W_1(\mu, \nu)$ is finite.