9 Exercise Sheet 9

Exercise 9.1. *Find the optimal transport map for the linear cost* $c(x, y) = |x - y|$ *between* $\mu = f \cdot \mathscr{L}^2$ *and* $\nu = g \cdot \mathscr{L}^2$ *in* $\mathbb{R}^2$*, where* $f(x) = \frac{1}{\pi}\chi_{B(0,1)}(x)$ *and* $g(x) = \frac{1}{8\pi}(4 - |x|^2)\chi_{B(0,2)}(x)$.

> **Hint:**
> Recall exercise 5.3 and use the potential $u(x) = -|x|$.

Exercise 9.2. *Let* $\mu = \frac{1}{\pi}\chi_{B(0,1)}\mathcal{L}^2$ *be the uniform probability measure on* $B(0,1) \subset \mathbb{R}^2$ *and let* $p_0 = (1, 0)$*,* $p_1 = (2, 0)$ *be two fixed points in* $\mathbb{R}^2$*. Describe the optimal transport map between* μ *and* $\frac{1}{2}(\delta_{p_0} + \delta_{p_1})$ *in the following two cases:*

(a) when the cost is $c(x, y) = \frac{1}{2}|x - y|^2$*;*

(b) when the cost is $c(x, y) = |x - y|$.

> **Hint:**
> Exploit that the graph of an optimal map is c-cyclically monotone.