

Probability Theory

Exercise sheet 7

Exercise 7.1 Let X and Y be two independent Bernoulli distributed random variables with parameter p . Define $Z = 1_{\{X+Y=0\}}$ and $\mathcal{G} = \sigma(Z)$. Find $E[X|\mathcal{G}]$ and $E[Y|\mathcal{G}]$. Are these random variables also independent?

Exercise 7.2 Let X and Y be random variables whose joint distribution is the uniform distribution on the triangle $\{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq x \leq 1\}$.

- Compute the distribution of Y/X .
- Show that Y/X and X are independent.
- Compute the conditional expectation $E[Y|X]$.

Exercise 7.3 Let X_1, X_2, \dots be independent random variables for which there exists a constant $M > 0$, such that $|X_n| \leq M$, P -a.s. for $n = 1, 2, \dots$. We write $S_n = X_1 + \dots + X_n$. Show that, if $\sum \text{Var}(X_n) = \infty$, then there exist constants a_n, b_n such that $(S_n - b_n)/a_n$ converges in distribution towards a standard normal random variable.

Submission deadline: 13:15, Nov 13

Location: During exercise class or in the tray outside of HG G53-54.

Class assignment:

| Students | Time & Date | Room | Assistant |
|----------|-------------|-----------|--------------------------|
| An-Gu | Tue 13-14 | HG F 26.5 | Daniel Balint |
| Ha-Lang | Tue 13-14 | ML H 41.1 | Daniel Contreras Salinas |
| Lanz-Sa | Tue 14-15 | HG F 26.5 | Daniel Balint |
| Sch-Zh | Tue 14-15 | ML H 41.1 | Chong Liu |

Office hours (Präsenz); Mon. and Thu., 12:00 - 13:00 in HG G32.6.

Exercise sheets and further information are also available on:

<http://metaphor.ethz.ch/x/2018/hs/401-3601-00L/>