Coordinator
Chong Liu

Probability Theory

Exercise sheet 9

Exercise 9.1 Let $(\Omega, \mathcal{F}, P)$ be a probability space with a filtration $(\mathcal{F}_n)_{n\geq 0}$. Let $S \leq T$ be two bounded $(\mathcal{F}_n)_{n\geq 0}$-stopping times and let $(X_n)_{n\geq 0}$ be an $(\mathcal{F}_n)_{n\geq 0}$-submartingale. Show that

$$E[X_T|\mathcal{F}_S] \geq X_S,\ P\text{-a.s..}$$

Exercise 9.2 Let $S, T : \Omega \to \mathbb{N} \cup \{\infty\}$ be $\mathcal{F}_n$-stopping times. Prove or provide a counter example disproving the following statements:

(a) $S - 1$ is a stopping time.

(b) $S + 1$ is a stopping time.

(c) $S \wedge T$ is a stopping time.

(d) $S \vee T$ is a stopping time.

(e) $S + T$ is a stopping time.

Exercise 9.3 Let Y_n, $n \geq 0$ be i.i.d. with $P[Y_0 = 1] = p$ and $P[Y_0 = 0] = 1 - p$ for some $p \in (0, 1)$. Let $\mathcal{F}_n := \sigma(Y_0, \ldots, Y_n)$ for $n \geq 0$ and define

$$T := \inf\{n \geq 0 \mid Y_n = 1\}.$$

Determine the Doob decomposition of $X_n := 1_{\{T\leq n\}}$, $n \geq 0$.
Hint: First check that X_n is an $\mathcal{F}_n$-submartingale. Then try to use Proposition 3.19.

Submission deadline: 13:15, Nov 27
Location: During exercise class or in the tray outside of HG G53–54.
Class assignment:

Students	Time & Date	Room	Assistant
An-Gu	Tue 13-14	HG F 26.5	Daniel Balint
Ha-Lang	Tue 13-14	ML H 41.1	Daniel Contreras Salinas
Lanz-Sa	Tue 14-15	HG F 26.5	Daniel Balint
Sch-Zh	Tue 14-15	ML H 41.1	Chong Liu

Office hours (Präsenz); Mon. and Thu., 12:00 - 13:00 in HG G32.6.
Exercise sheets and further information are also available on:
http://metaphor.ethz.ch/x/2018/hs/401-3601-00L/