

## Probability Theory

### Self evaluation quiz, November 08

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Number:

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1. State Kolmogorov's 0-1 law. Give an example where it applies.
2. State the Continuity Theorem.  
If  $\mu_n$  is the centered normal distribution with variance  $n$ , does the sequence  $\mu_n$ ,  $n \geq 1$ , converge weakly? Justify your answer.
3. State the Three Series Theorem.  
If  $X_k = \frac{1}{k^\alpha} + \frac{1}{k}Z_k$ ,  $k \geq 1$ , where  $Z_k$ ,  $k \geq 1$  are i.i.d. with  $P[Z_k = 1] = P[Z_k = -1] = \frac{1}{2}$  and  $\alpha > 0$ , discuss the convergence property (in the almost surely sense) of the random series  $\sum_{k \geq 1} X_k$  (depending on the value of  $\alpha$ ).
4. State the Lindeberg-Feller Theorem. Show that it implies the Central Limit Theorem.











