SHEET 4.

Exercise 1 Let $\beta \geqslant 0$

Prove that $m(\beta, h) \cdot \begin{cases} > 0 & \text{if } h > 0 \\ < 0 & \text{if } h < 0 \end{cases}$

What happens for $h = 0$?

Exercise 2

Let $A, B \subset \Lambda \subset\subset \mathbb{Z}^d$. $h \geqslant 0$

Prove that $\mu^{+}_{\Lambda, h}$, $\underset{\substack{\uparrow \\ \text{defined by } \omega = 0 \text{ at } \partial\Lambda}}{\mu^{\emptyset}_{\Lambda, h}}$ satisfy the

GKS-inequalities :

$$\langle \sigma_A \rangle \geqslant 0 \quad \text{and} \quad \langle \sigma_A \sigma_B \rangle \geqslant \langle \sigma_A \rangle \langle \sigma_B \rangle$$

Deduce that μ^{+}_{h} (the infinite volume measure) also satisfies the GKS inequalities.

Exercise 3.

(i) Prove that μ^+ satisfy the mixing property:

$$\forall\, E, F \text{ events} \qquad \lim_{|x|\to\infty} \mu^+\left[E \cap \Theta_x^{-1} F\right] = \mu^+[E]\,\mu^+[F].$$

(ii) Deduce that μ^+ is ergodic: for every translation invariant event E (i.e. $\Theta_x^{-1} E = E \ \forall x$).

we have $\mu^+[E] \in \{0,1\}$.

(iii) Use the mixing property to show that

$$\frac{1}{|B_n|} \sum_{x \in B_n} \sigma_x \xrightarrow[n\to\infty]{} m(\beta, h) \quad \text{in probability.}$$

Exercise 4

Without using exercise 3, Prove that

$$m(\beta, h) = \lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x \in B_n} \langle \sigma_x \rangle_{B_n}^+ .$$