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Non-Life Insurance: Mathematics and Statistics

Exercise sheet 12

Exercise 12.1 Chain-Ladder Algorithm
We write $i = 1, \ldots, I$ for the accident years denoting the years of claims occurrence. For every accident year we consider development years $j = 0, \ldots, J$. For all $i = 1, \ldots, I$ and $j = 0, \ldots, J$ we write $C_{i,j}$ for the cumulative payments up to development year j for all claims that have occurred in accident year i. For simplicity, we set $I = J + 1 = 10$. Assume that we have observations

$$\mathcal{D}_I = \{C_{i,j} \,|\, i + j \leq I,\, 1 \leq i \leq I,\, 0 \leq j \leq J\}$$

given by the following upper claims reserving triangle:

accident	development year j									
year i	0	1	2	3	4	5	6	7	8	9
1	5’946’975	9’668’212	10’563’929	10’771’690	10’978’394	11’040’518	11’106’331	11’121’181	11’132’310	11’148’124
2	6’346’756	9’593’162	10’316’383	10’468’180	10’536’004	10’572’608	10’625’360	10’636’546	10’648’192	
3	6’269’090	9’245’313	10’092’366	10’355’134	10’507’837	10’573’282	10’626’827	10’635’751		
4	5’863’015	8’546’239	9’268’771	9’459’424	9’592’399	9’680’740	9’724’068			
5	5’778’885	8’524’114	9’178’009	9’451’404	9’681’692	9’786’916				
6	6’1847’93	9’013’132	9’585’897	9’830’796	9’935’753					
7	5’600’184	8’493’391	9’056’505	9’282’022						
8	5’288’066	7’728’169	8’256’211							
9	5’290’793	7’648’729								
10	5’675’568									

Table 1: Upper claims reserving triangle $\mathcal{D}_I$.

This data set can be downloaded from

https://people.math.ethz.ch/~wueth/exercises2.html

by clicking on “Data to the Examples”.

(a) Use the chain-ladder (CL) method to predict the lower triangle

$$\mathcal{D}_I^c = \{C_{i,j} \,|\, i + j > I,\, 1 \leq i \leq I,\, 0 \leq j \leq J\}.$$

(b) Calculate the CL reserves $\widehat{\mathcal{R}}_i^{CL}$ for all accident years $i \in \{1, \ldots, I\}$.

Exercise 12.2 Bornhuetter-Ferguson Algorithm
Consider the same setup as in Exercise 12.1.

(a) We assume that we have prior informations $\widehat{\mu}_1, \ldots, \widehat{\mu}_I$ for the expected ultimate claims $\mathbb{E}[C_{1,J}], \ldots, \mathbb{E}[C_{I,J}]$ given by

accident year i	1	2	3	4	5
prior information $\widehat{\mu}_i$	11’653’101	11’367’306	10’962’965	10’616’762	11’044’881

accident year i	6	7	8	9	10
prior information $\widehat{\mu}_i$	11’480’700	11’413’572	11’126’527	10’986’548	11’618’437

Table 2: Prior informations $\widehat{\mu}_1, \ldots, \widehat{\mu}_I$.

Use the Bornhuetter-Ferguson (BF) method to calculate the BF reserves $\widehat{\mathcal{R}}_i^{BF}$ for all accident years $i \in \{1, \dots, I\}$.

(b) Explain why in this example we have, for all accident years $i \in \{2, \dots, I\}$,

$$\widehat{\mathcal{R}}_i^{CL} < \widehat{\mathcal{R}}_i^{BF},$$

where $\widehat{\mathcal{R}}_i^{CL}$ denotes the CL reserves for accident year i calculated in Exercise 12.1.

Exercise 12.3 Mack's Formula and Merz-Wüthrich (MW) Formula (R Exercise)
Consider the same setup as in Exercise 12.1.

(a) Write an R code using the R package `ChainLadder` in order to calculate the conditional mean square error of prediction

$$\mathrm{msep}^{\mathrm{Mack}}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}^{CL}_{i,J}\right),$$

given in formula (9.21) of the lecture notes, for all accident years $i \in \{1, \dots, I\}$. Moreover, calculate the conditional mean square error of prediction for aggregated accident years

$$\mathrm{msep}^{\mathrm{Mack}}_{\sum_{i=1}^I C_{i,J}|\mathcal{D}_I}\left(\sum_{i=1}^I \widehat{C}^{CL}_{i,J}\right),$$

given in formula (9.22) of the lecture notes. Interpret the square-rooted conditional mean square errors of prediction relative to the claims reserves calculated in Exercise 12.1.

(b) Determine the one-year (run-off) uncertainty

$$\mathrm{msep}^{\mathrm{MW}}_{\mathrm{CDR}_{i,I+1}|\mathcal{D}_I}(0),$$

given in formula (9.34) of the lecture notes, for all accident years $i \in \{1, \dots, I\}$. Moreover, determine the one-year (run-off) uncertainty for aggregated accident years

$$\mathrm{msep}^{\mathrm{MW}}_{\sum_{i=1}^I \mathrm{CDR}_{i,I+1}|\mathcal{D}_I}(0),$$

given in formula (9.35) of the lecture notes. Interpret the square-rooted one-year (run-off) uncertainties relative to the square-rooted conditional mean square errors of prediction calculated in part (a).

Exercise 12.4 Conditional MSEP and Claims Development Result
In the first part of this exercise we show that the conditional mean square error of prediction can be decoupled into the process uncertainty and the parameter estimation error. In the second part we show that the claims development results are uncorrelated.

(a) Let $\mathcal{D}$ be a σ-algebra, $\widehat{X}$ a $\mathcal{D}$-measurable predictor for the random variable X and both $\widehat{X}$ and X square-integrable. Show that

$$\mathrm{msep}_{X|\mathcal{D}}\left(\widehat{X}\right) = \mathbb{E}\left[\left(X - \widehat{X}\right)^2 \middle| \mathcal{D}\right] = \mathrm{Var}(X|\mathcal{D}) + \left(\mathbb{E}[X|\mathcal{D}] - \widehat{X}\right)^2 \quad \text{a.s.}$$

(b) Suppose that we are in the framework of Section 9.4.1 of the lecture notes. Let $t_1 < t_2 \in \mathbb{N}$ such that $t_1 \geq I$ and $i > t_2 - J$ and assume that $C_{i,J}$ has finite second moment. Show that

$$\mathrm{Cov}\left(\mathrm{CDR}_{i,t_1+1}, \mathrm{CDR}_{i,t_2+1}\right) = 0.$$