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Non-Life Insurance: Mathematics and Statistics

Exercise sheet 3

Exercise 3.1 No-Claims Bonus
An insurance company decides to offer a no-claims bonus to good car drivers, namely

- a 10% discount on the premium after three years of no claim, and
- a 20% discount on the premium after six years of no claim.

How does the premium need to be adjusted such that the premium income remains the same as before the grant of the no-claims bonus? For simplicity, we consider one car driver who has been insured for at least six years. Answer the question in the following two situations:

(a) The claim counts of the individual years of the considered car driver are independent, identically Poisson distributed random variables with frequency parameter $\lambda = 0.2$.

(b) Suppose Θ follows a gamma distribution with shape parameter $\gamma = 1$ and scale parameter $c = 1$, i.e. the density f_Θ of Θ is given by

$$f_\Theta(x) = \begin{cases} \exp\{-x\}, & \text{if } x \geq 0, \\ 0, & \text{else.} \end{cases}$$

Now, conditionally given Θ, the claim counts of the individual years of the considered car driver are independent, identically Poisson distributed random variables with frequency parameter $\Theta\lambda$, where $\lambda = 0.2$ as above.

Exercise 3.2 Claims Count Distribution
Suppose that in a given line of business of an insurance company the numbers of claims of the last ten years are modeled by random variables $N_1, \ldots, N_{10}$. We assume that $N_1, \ldots, N_{10}$ are i.i.d. and that we have collected the following observations:

t	1	2	3	4	5	6	7	8	9	10
N_t	7	21	19	18	25	17	33	6	39	28

Table 1: Observed number of claims N_t over the last ten years.

If you had to choose between a binomial distribution, a Poisson distribution and a negative binomial distribution for modeling the number of claims, which distribution would you prefer? Give a short argument.

Exercise 3.3 Central Limit Theorem
Let n be the number of claims and $Y_1, \ldots, Y_n$ the corresponding claim sizes, where we assume that $Y_1, \ldots, Y_n$ are independent, identically distributed random variables with expectation $\mathbb{E}[Y_1] = \mu$ and coefficient of variation $\mathrm{Vco}(Y_1) = 4$. Use the Central Limit Theorem to determine an approximate minimum number of claims such that with probability of at least 95% the deviation of the empirical mean $\frac{1}{n}\sum_{i=1}^{n} Y_i$ from μ is less than 1%.

Exercise 3.4 Compound Binomial Distribution
Assume $S \sim \text{CompBinom}(v, p, G)$ for given $v \in \mathbb{N}$, $p \in (0,1)$ and individual claim size distribution G. Let $M > 0$ such that $G(M) \in (0,1)$. Define the compound distributions of claims Y_i that are at most of size M resp. that exceed the threshold M by

$$S_{\text{sc}} = \sum_{i=1}^{N} Y_i \; 1_{\{Y_i \leq M\}} \qquad \text{resp.} \qquad S_{\text{lc}} = \sum_{i=1}^{N} Y_i \; 1_{\{Y_i > M\}}.$$

(a) Show that $S_{\text{lc}} \sim \text{CompBinom}(v, p[1 - G(M)], G_{\text{lc}})$, where the large claims size distribution function G_{lc} satisfies $G_{\text{lc}}(y) = \mathbb{P}[Y_1 \leq y | Y_1 > M]$.

(b) Give a short argument which shows that S_{sc} and S_{lc} are not independent.