Coordinator
Andrea Gabrielli

Non-Life Insurance: Mathematics and Statistics

Exercise sheet 6

Exercise 6.1 Goodness-of-Fit Test
Suppose we are given the following claim size data (in increasing order) coming from independent realizations of an unknown claim size distribution:

210, 215, 228, 232, 303, 327, 344, 360, 365, 379, 402, 413, 437, 481, 521, 593, 611, 677, 910, 1623.

Use the intervals

$$I_1 = [200, 239), \quad I_2 = [239, 301), \quad I_3 = [301, 416), \quad I_4 = [416, 725), \quad I_5 = [725, +\infty)$$

to perform a χ^2-goodness-of-fit test at significance level of 5% to test the null hypothesis of having a Pareto distribution with threshold $\theta = 200$ and tail index $\alpha = 1.25$ as claim size distribution.

Exercise 6.2 Log-Normal Distribution and Deductible
Assume that the total claim amount

$$S = \sum_{i=1}^{N} Y_i$$

in a given line of business has a compound distribution with $\mathbb{E}[N] = \lambda v$, where λ denotes the claims frequency, and with a log-normal distribution with mean parameter $\mu \in \mathbb{R}$ and variance parameter $\sigma^2 > 0$ as claim size distribution.

(a) Show that

$$\begin{aligned} \mathbb{E}[Y_1] &= \exp\left\{\mu + \frac{\sigma^2}{2}\right\}, \\ \mathrm{Var}(Y_1) &= \exp\left\{2\mu + \sigma^2\right\}\left(\exp\left\{\sigma^2\right\} - 1\right) \quad \text{and} \\ \mathrm{Vco}(Y_1) &= \sqrt{\exp\left\{\sigma^2\right\} - 1}. \end{aligned}$$

Hint: Use the moment generating function of a Gaussian distribution with mean μ and variance σ^2.

(b) Suppose that $\mathbb{E}[Y_1] = 3'000$ and $\mathrm{Vco}(Y_1) = 4$. Up to now, the insurance company was not offering contracts with deductibles. Now it wants to offer a deductible of $d = 500$. Answer the following questions:

(i) How does the claims frequency λ change by the introduction of the deductible?

(ii) How does the expected claim size $\mathbb{E}[Y_1]$ change by the introduction of the deductible?

(iii) How does the expected total claim amount $\mathbb{E}[S]$ change by the introduction of the deductible?

Exercise 6.3 Re-Insurance Covers and Leverage Effect
In Figure 1 we display the distribution function of a loss $Y \sim \Gamma(1, \frac{1}{400})$ and the distribution function of the loss after applying different re-insurance covers to Y.

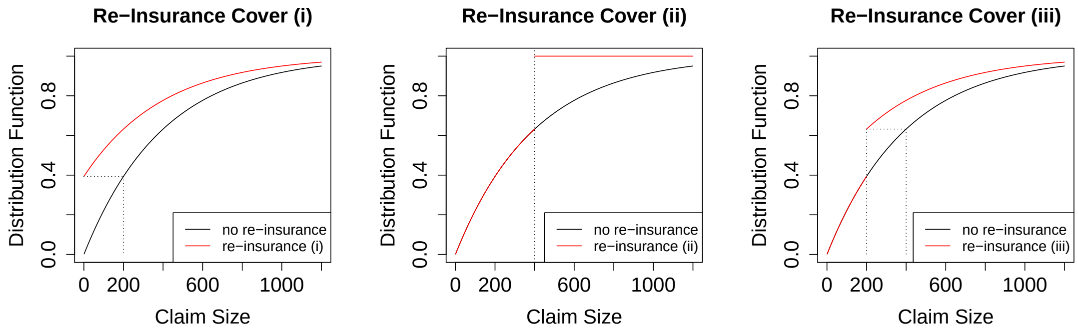

Figure 1: Distribution functions implied by re-insurance contracts (i), (ii) and (iii).

(a) Let $d > 0$. Show that Y satisfies

$$\mathbb{E}[(Y - d)_+] = \mathbb{P}[Y > d]\mathbb{E}[Y].$$

(b) Can you explicitly determine the re-insurance covers from the graphs in Figure 1?

(c) Calculate the expected values of these modified contracts.

(d) Assume that a first claim Y_0 has the same distribution as Y, and that a second claim Y_1 fulfills $Y_1 \overset{(d)}{=} (1 + i)Y_0$, for a constant inflation rate $i > 0$. Let $d > 0$. Show the leverage effect

$$\mathbb{E}[(Y_1 - d)_+] > (1 + i)\mathbb{E}[(Y_0 - d)_+].$$

Give an appropriate explanation for this leverage effect.

Exercise 6.4 Inflation and Deductible
This year's claims in a storm insurance portfolio have been modeled by a Pareto distribution with threshold $\theta > 0$ and tail index $\alpha > 1$. The threshold θ (in CHF) can be understood as deductible. Suppose that the inflation for next year is expected to be $100 \cdot r\,\%$ for some $r > 0$. By how much do we have to increase the deductible θ next year such that the average claim payment remains unchanged?