Non-Life Insurance: Mathematics and Statistics

Exercise sheet 6

Exercise 6.1 Goodness-of-Fit Test
Suppose we are given the following claim size data (in increasing order) coming from independent realizations of an unknown claim size distribution:


Use the intervals

\[ I_1 = [200, 239), \quad I_2 = [239, 301), \quad I_3 = [301, 416), \quad I_4 = [416, 725), \quad I_5 = [725, +\infty) \]

performed a \( \chi^2 \)-goodness-of-fit test at significance level of 5% to test the null hypothesis of having a Pareto distribution with threshold \( \theta = 200 \) and tail index \( \alpha = 1.25 \) as claim size distribution.

Exercise 6.2 Log-Normal Distribution and Deductible
Assume that the total claim amount

\[ S = \sum_{i=1}^{N} Y_i \]

in a given line of business has a compound distribution with \( \mathbb{E}[N] = \lambda v \), where \( \lambda \) denotes the claims frequency, and with a log-normal distribution with mean parameter \( \mu \in \mathbb{R} \) and variance parameter \( \sigma^2 > 0 \) as claim size distribution.

(a) Show that

\[ \mathbb{E}[Y_1] = \exp \left( \mu + \frac{\sigma^2}{2} \right), \]

\[ \text{Var}(Y_1) = \exp \left( 2\mu + \sigma^2 \right) \left( \exp \{ \sigma^2 \} - 1 \right) \quad \text{and} \]

\[ \text{Vco}(Y_1) = \sqrt{\exp \{ \sigma^2 \} - 1}. \]

Hint: Use the moment generating function of a Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \).

(b) Suppose that \( \mathbb{E}[Y_1] = 3'000 \) and \( \text{Vco}(Y_1) = 4 \). Up to now, the insurance company was not offering contracts with deductibles. Now it wants to offer a deductible of \( d = 500 \). Answer the following questions:

(i) How does the claims frequency \( \lambda \) change by the introduction of the deductible?
(ii) How does the expected claim size \( \mathbb{E}[Y_1] \) change by the introduction of the deductible?
(iii) How does the expected total claim amount \( \mathbb{E}[S] \) change by the introduction of the deductible?
Exercise 6.3 Re-Insurance Covers and Leverage Effect

In Figure 1 we display the distribution function of a loss $Y \sim \Gamma(1, \frac{1}{400})$ and the distribution function of the loss after applying different re-insurance covers to $Y$.

![Distribution functions implied by re-insurance contracts (i), (ii) and (iii).](image)

(a) Let $d > 0$. Show that $Y$ satisfies

$$
$$

(b) Can you explicitly determine the re-insurance covers from the graphs in Figure 1?

(c) Calculate the expected values of these modified contracts.

(d) Assume that a first claim $Y_0$ has the same distribution as $Y$, and that a second claim $Y_1$ fulfills $Y_1 \overset{(d)}{=} (1 + i)Y_0$, for a constant inflation rate $i > 0$. Let $d > 0$. Show the leverage effect

$$
E[(Y_1 - d)_+] > (1 + i)E[(Y_0 - d)_+].
$$

Give an appropriate explanation for this leverage effect.

Exercise 6.4 Inflation and Deductible

This year’s claims in a storm insurance portfolio have been modeled by a Pareto distribution with threshold $\theta > 0$ and tail index $\alpha > 1$. The threshold $\theta$ (in CHF) can be understood as deductible. Suppose that the inflation for next year is expected to be $100 \cdot r \%$ for some $r > 0$. By how much do we have to increase the deductible $\theta$ next year such that the average claim payment remains unchanged?