# Non-Life Insurance: Mathematics and Statistics

## Exercise sheet 8

**Exercise 8.1 Panjer Algorithm**
In this exercise we use the Panjer algorithm to calculate monthly health insurance premiums for different franchises $d$. We assume that the yearly claim amount

$$S = \sum_{i=1}^{N} Y_i$$

of a given customer is compound Poisson distributed with $N \sim \text{Poi}(1)$ and $Y_1 \overset{(d)}{=} k + Z$, where $k = 100$ CHF and $Z \sim \text{LN}(\mu = 7.8, \sigma^2 = 1)$. In health insurance the policyholder can choose between different franchises $d \in \{300, 500, 1'000, 1'500, 2'000, 2'500\}$. The franchise $d$ describes the threshold up to which the policyholder has to pay everything by himself. Moreover, the policyholder has to pay $\alpha = 10\%$ of the part of the total claim amount $S$ that exceeds the franchise $d$, but only up to a maximal amount of $M = 700$ CHF. Thus, the yearly amount paid by the customer is given by

$$S_{\text{ins}} = \min\{S, d\} + \min\{\alpha \cdot (S - d)_+, M\}.$$

If we define $\pi_0 = \mathbb{E}[S]$ and $\pi_{\text{ins}} = \mathbb{E}[S_{\text{ins}}]$, the monthly pure risk premium $\pi$ is given by

$$\pi = \frac{\pi_0 - \pi_{\text{ins}}}{12}.$$

Calculate $\pi$ for the different franchises $d \in \{300, 500, 1'000, 1'500, 2'000, 2'500\}$ using the Panjer algorithm. In order to do that, discretize the translated log-normal distribution using a span of $s = 10$ and putting all the probability mass to the upper end of the intervals.

**Exercise 8.2 Fast Fourier Transform (R Exercise)**
Consider the same setup as in Exercise 7.2 and 7.3. In particular, assume that $S$ has a compound Poisson distribution with expected number of claims $\lambda v = 1'000$ and claim sizes following a gamma distribution with shape parameter $\gamma = 100$ and scale parameter $c = \frac{1}{10}$. Write an R code that applies the fast Fourier transform using a threshold of $n = 2'000'000$ in order to determine the distribution function of $S$. Compare the resulting distribution function to the distribution function found in Exercise 7.3, where we used Monte Carlo simulations. Moreover, determine the 0.95-quantile $q_{0.95}$ and the 0.99-quantile $q_{0.99}$ of S and compare them to the values found in Exercise 7.2.

**Exercise 8.3 Variance Loading Principle**
We would like to insure the following car fleet:

| $i$ | $v_i$ | $\lambda_i$ | $\mathbb{E}[Y_1^{(i)}]$ | $\text{Vco}(Y_1^{(i)})$ |
|---|---|---|---|---|
| passenger car | 40 | 25% | 2'000 | 2.5 |
| delivery van | 30 | 23% | 1'700 | 2.0 |
| truck | 10 | 19% | 4'000 | 3.0 |

Table 1: Volumes, claim frequencies, expected claim sizes and coefficients of variation of the claim sizes for the three sections of the car fleet.

Assume that the total claim amounts for passenger cars, delivery vans and trucks can be modeled by independent compound Poisson distributions.

(a) Calculate the expected claim amount of the car fleet.

(b) Calculate the premium for the car fleet using the variance loading principle with $\alpha = 3 \cdot 10^{-6}$.

**Exercise 8.4 Panjer Distribution**

Let $N$ be a random variable that has a Panjer distribution with parameters $a, b \in \mathbb{R}$. Calculate $\mathbb{E}[N]$ and $\mathrm{Var}(N)$. What can you say about the ratio of $\mathrm{Var}(N)$ to $\mathbb{E}[N]$?