Mathematical Finance

Exercise sheet 5

Please hand in your solutions by Wednesday, 07.11.2018, 12:00 into your assistant's box next to HG G 53.2.

Exercise 5.1 Construct an example where S is a martingale but $\mathcal{G}_{\mathrm{adm}} = \{0\}$.

Exercise 5.2 Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a filtered probability space, $S = (S^1, \dots, S_t^d)_{t\in[0,T]}$ a d-dimensional semimartingale and $Q \approx P$ on $\mathcal{F}_T$ an equivalent probability measure.

(a) Assume that $\mathcal{F}_0$ is trivial and Q is a separating measure for S. Show that if S is (locally) bounded, then Q is an equivalent (local) martingale measure for S.

(b) Assume that Q is an equivalent σ-martingale measure for S. Show that it is also an equivalent separating measure.

(c) Now assume that $d = 1$, that $(\mathcal{F}_t)_{t\in[0,T]}$ is the natural (completed) filtration of S and that the process $S = (S_t)_{t\in[0,T]}$ is of the form

$$S_t = \begin{cases} 0 & \text{for } 0 \le t < T, \\ X & \text{for } t = T, \end{cases}$$

where X is normally distributed with mean $\mu \neq 0$ and variance $\sigma^2 > 0$. Show that in this case, the class $\mathcal{M}_{\mathrm{sep}}$ of equivalent separating measures for S is strictly bigger than the class $\mathcal{M}_\sigma$ of equivalent σ-martingale measures for S.

Exercise 5.3 Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\ge 0}, P)$ be a filtered probability space and $X = (X_t)_{t\ge 0}$ a d-dimensional RCLL σ-martingale with $X_0 = 0$.

(a) Prove that there exists a sequence of predictable sets $(\Sigma_n)_{n\ge 1}$ (i.e. each Σ_n lies in the predictable σ-algebra $\mathcal{P}$ on $\Omega \times [0, \infty)$) such that $\Sigma_n \subseteq \Sigma_{n+1}$ for all n, $\bigcup_n \Sigma_n = \Omega \times [0, \infty)$ and for each n, the process $1_{\Sigma_n} \bullet X = (1_{\Sigma_n} \bullet X^i)_{i=1,\dots,d}$ is a d-dimensional uniformly integrable martingale.

(b) Show that every continuous σ-martingale is a local martingale.