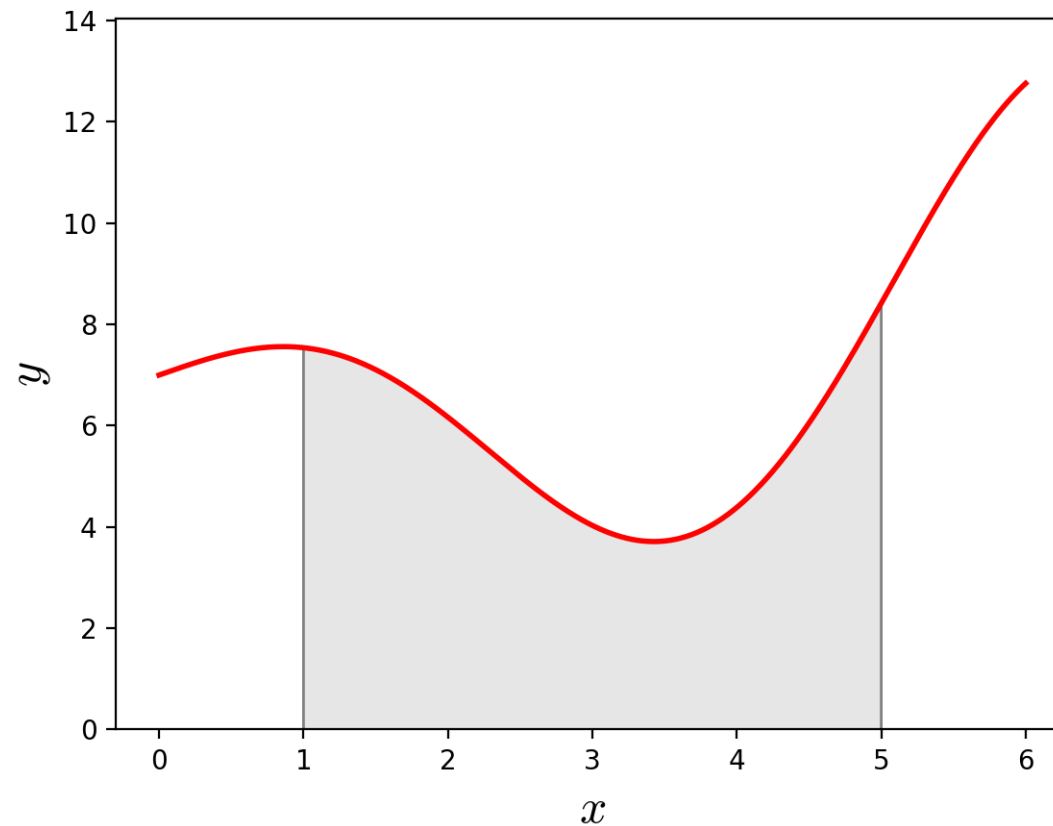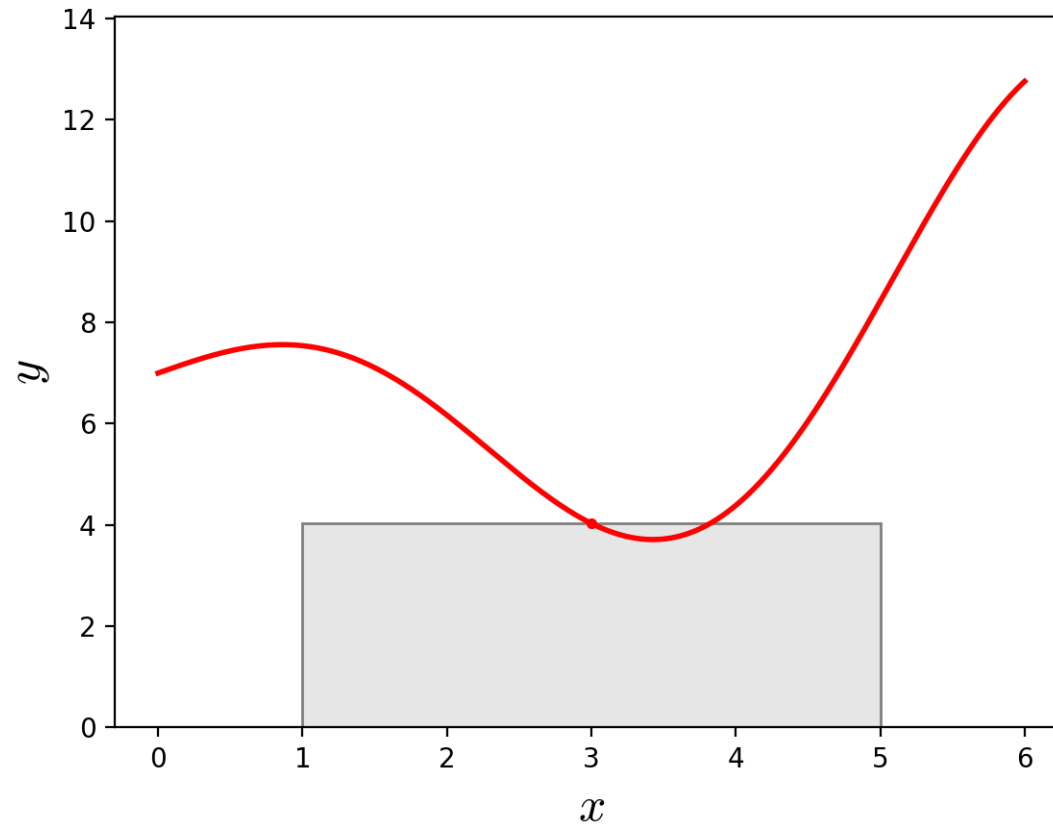# Quadratur-Regeln

$$f(x) = x\cos(x) + 7$$

$$I[f] = \int_1^5 f(x)\mathrm{d}x = 22.1073...$$

Mittelpunktsregel

$$f(x) = x \cos(x) + 7$$
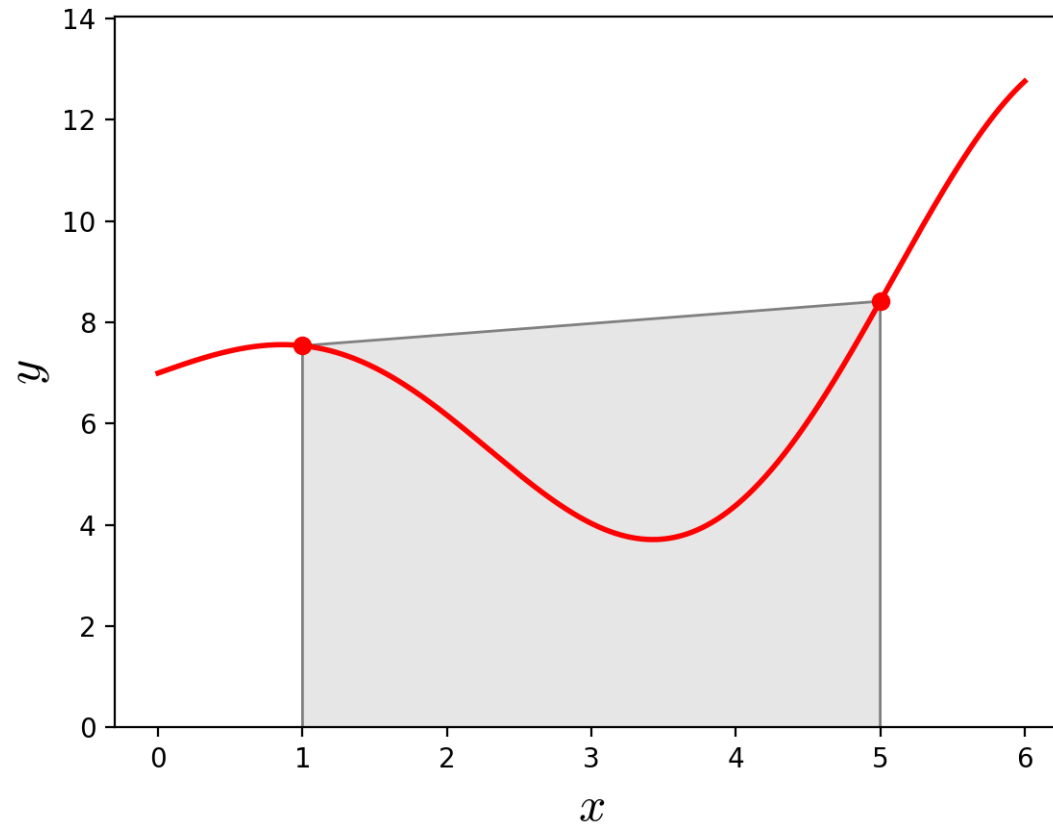

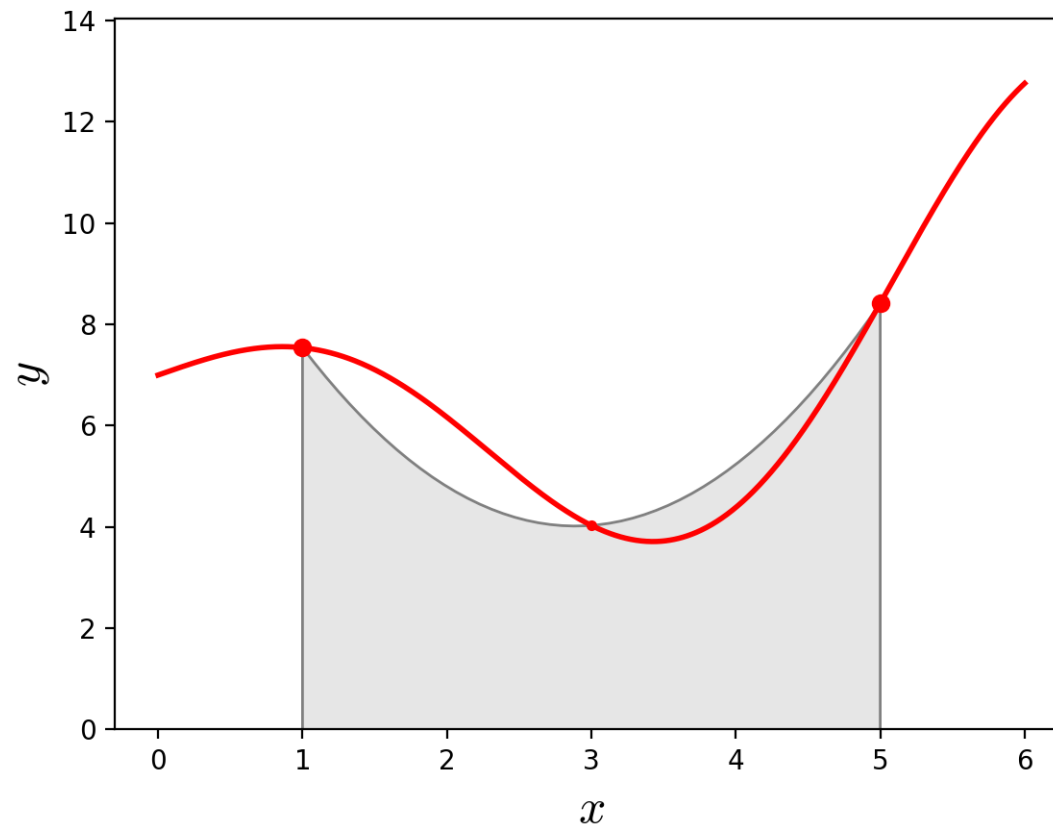$$I[f] = 22.1073... \quad Q_0[f] = 16.1201... \quad E[f] = 5.9872...$$

Trapezsregel

$$f(x) = x\cos(x) + 7$$

$$I[f] = 22.1073... \quad Q_1[f] = 31.9172... \quad E[f] = 9.8100...$$

Simpson-Regel

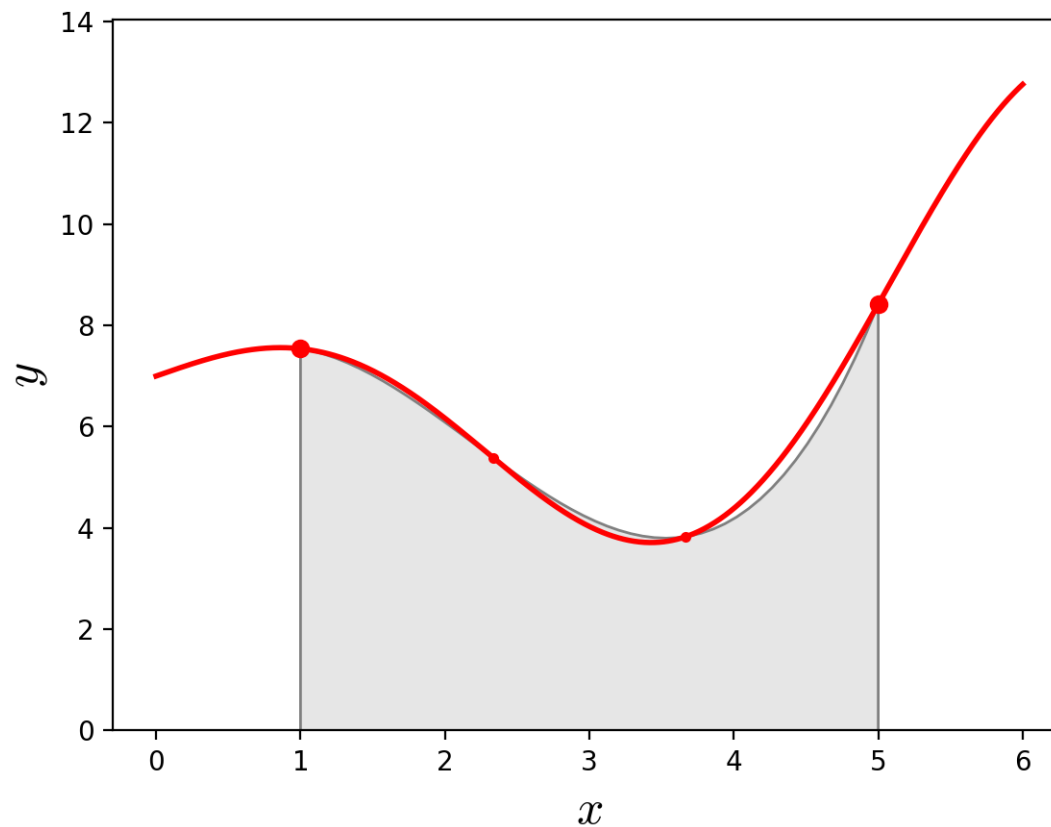$$f(x) = x\cos(x) + 7$$

$$I[f] = 22.1073... \quad Q_2[f] = 21.3858... \quad E[f] = 0.7215...$$
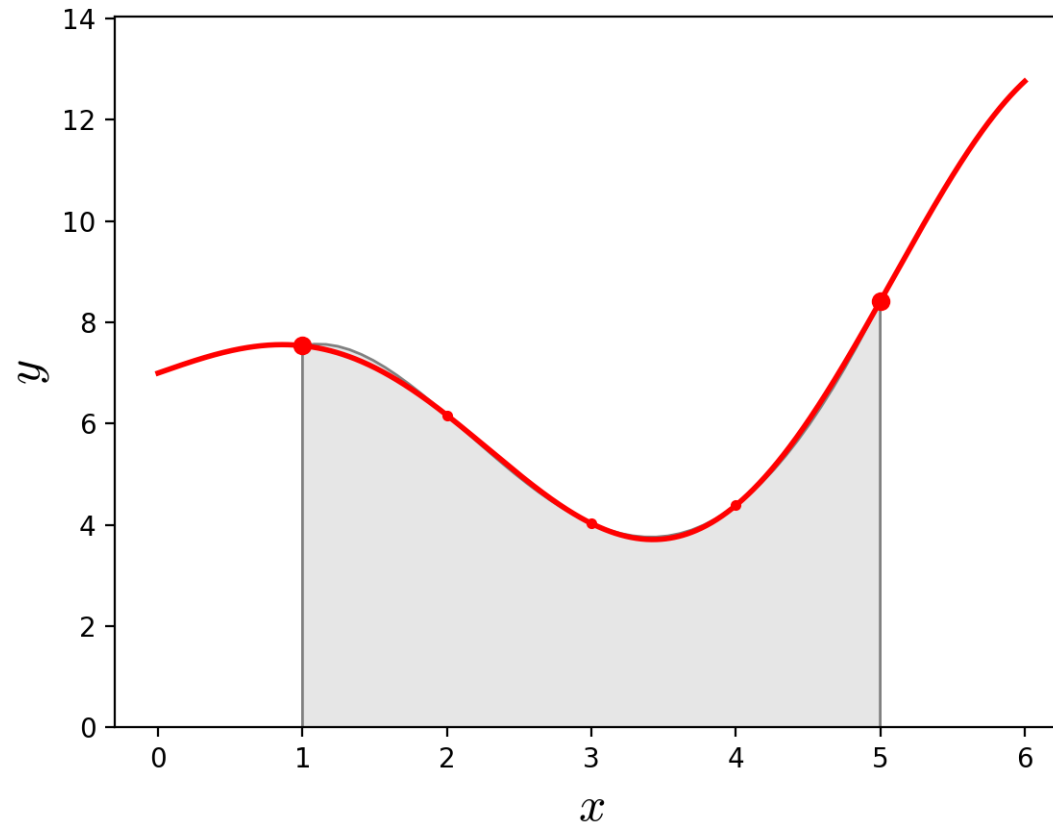
3/8-Regel

$$f(x) = x \cos(x) + 7$$

$$I[f] = 22.1073... \qquad Q_3[f] = 21.8026... \qquad E[f] = 0.3047...$$

Milne-Regel

$$f(x) = x\cos(x) + 7$$
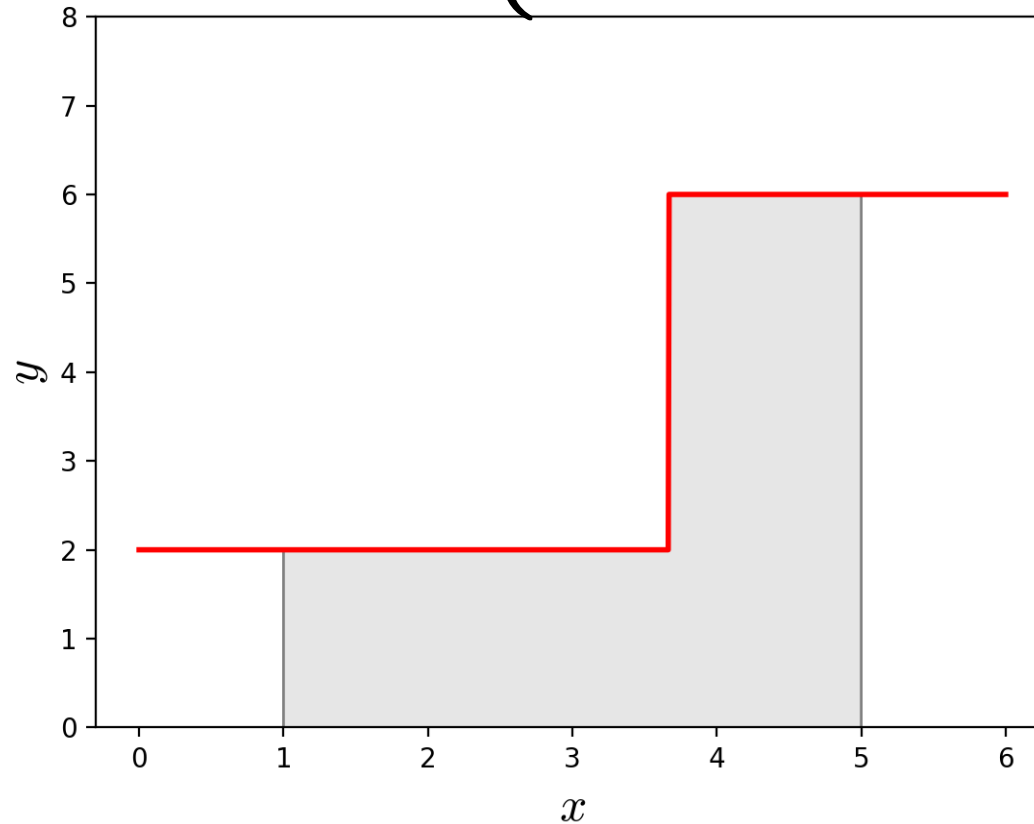

$$I[f] = 22.1073... \quad Q_4[f] = 22.1231... \quad E[f] = 0.0159...$$

Bsp.: (9)

Quadrature

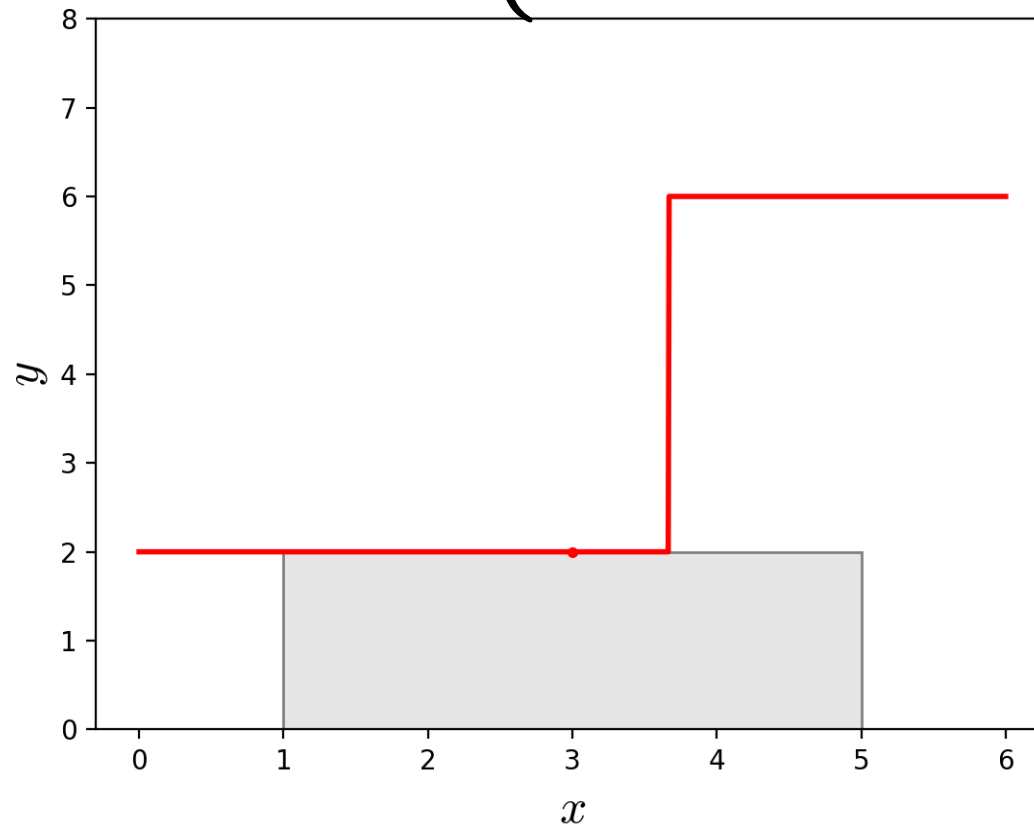$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

$$I[f] = \int_1^5 f(x)\mathrm{d}x = 15.4336...$$

Bsp.: (9)

Mittelpunktsregel

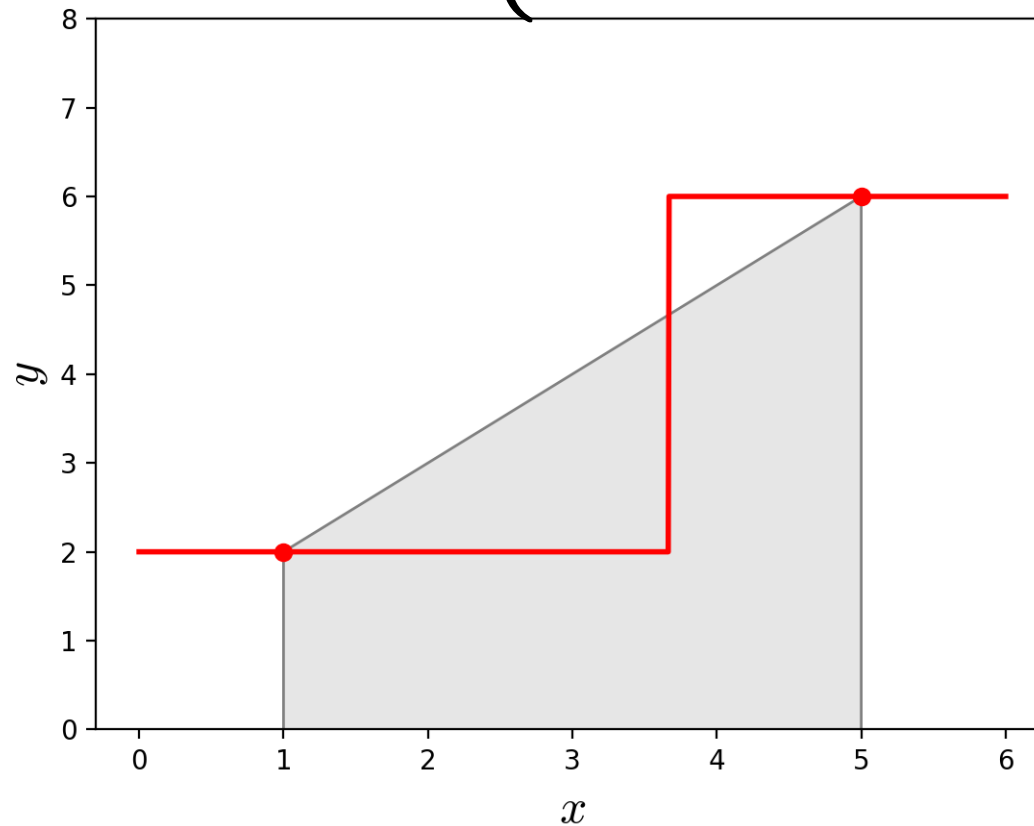$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

$$I[f] = 15.4336... \quad Q_0[f] = 8 \quad E[f] = 7.4336...$$

Trapezregel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$
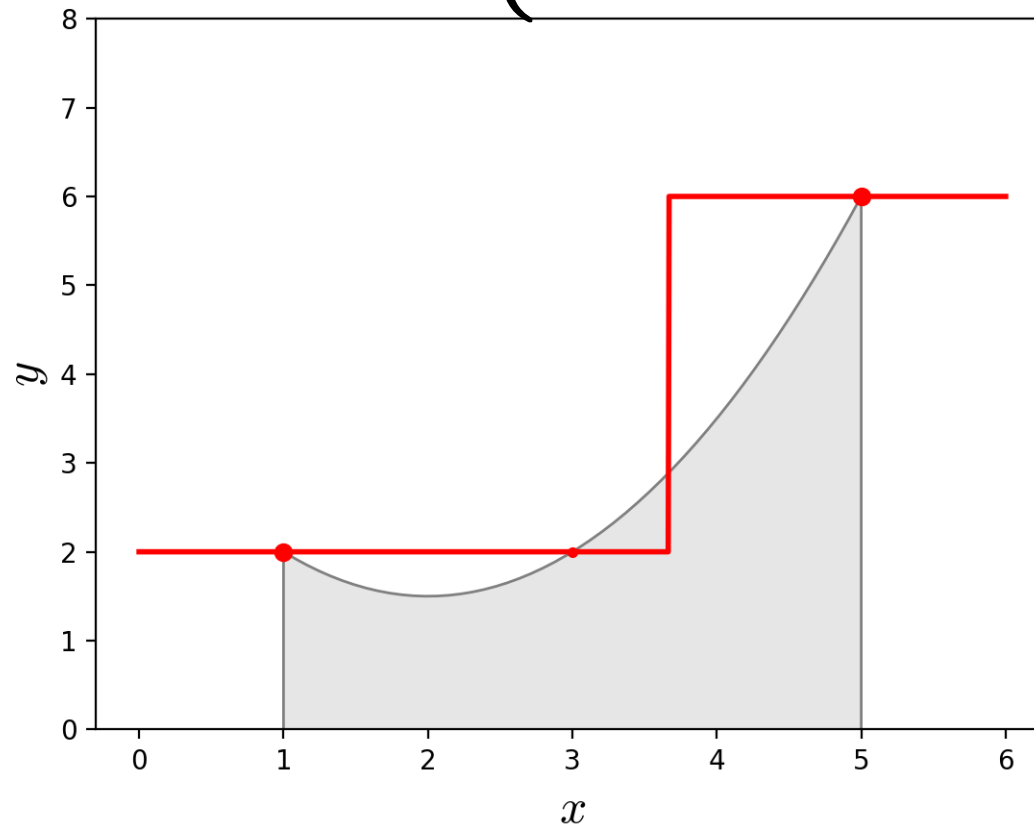

$$I[f] = 15.4336... \qquad Q_1[f] = 16 \qquad E[f] = 0.5664...$$

Bsp.: (9)

Simpson-Regel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

$$I[f] = 15.4336... \quad Q_2[f] = 10.6666... \quad E[f] = 4.7670...$$

3/8-Regel
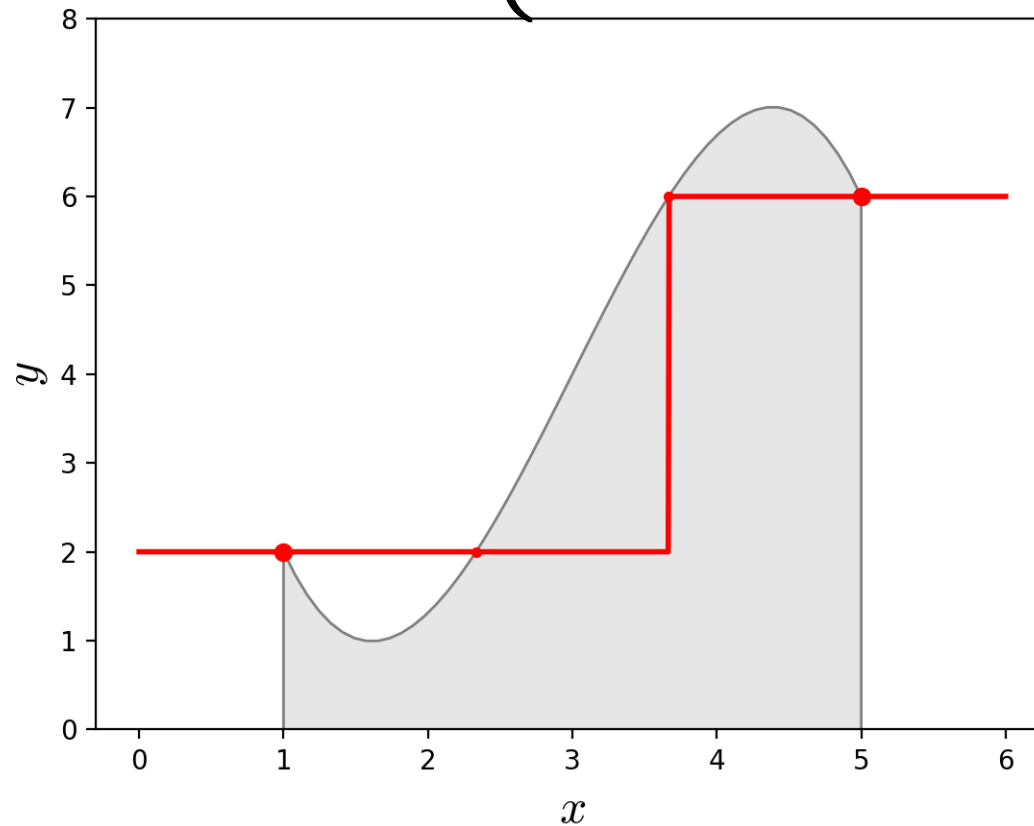
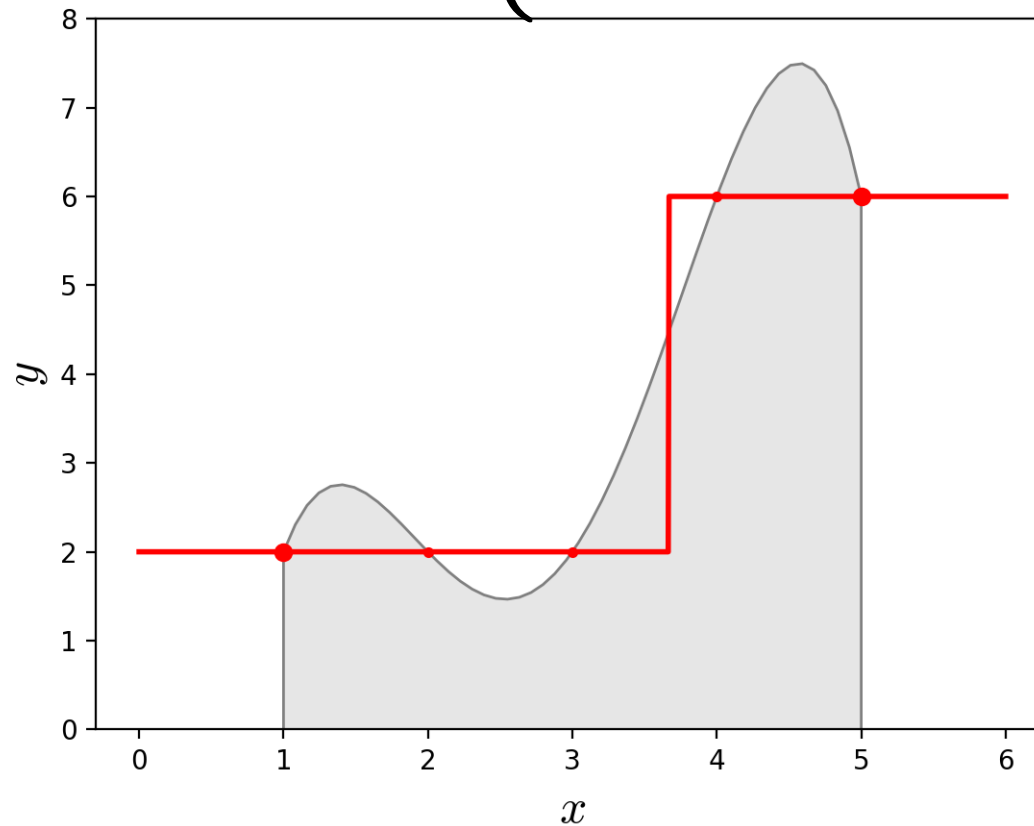$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

$$I[f] = 15.4336... \qquad Q_3[f] = 16 \qquad E[f] = 0.5664...$$

Bsp.: (9)

Milne-Regel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$
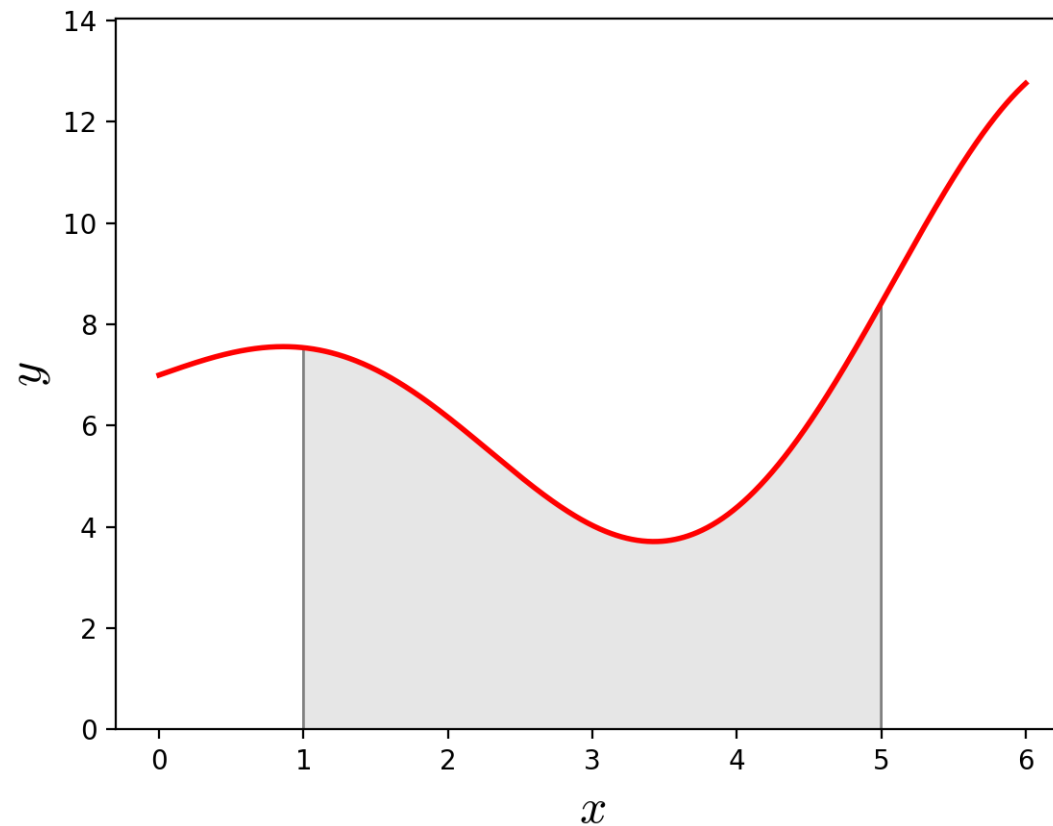

$$I[f] = 15.4336... \qquad Q_4[f] = 14.9333... \qquad E[f] = 0.5003...$$

Summierte Quadratur-Regeln

$$f(x) = x\cos(x) + 7$$

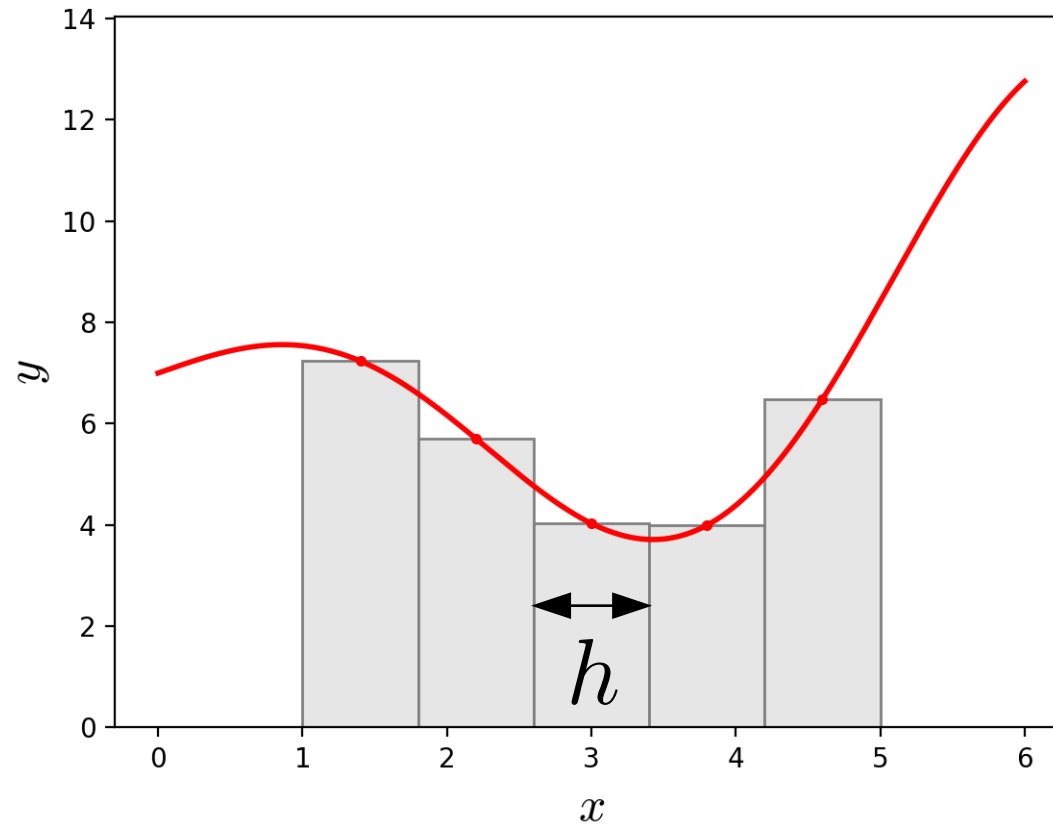$$I[f] = \int_1^5 f(x)\mathrm{d}x = 22.1073...$$

Summierte Mittelpunktsregel

$$f(x) = x\cos(x) + 7$$
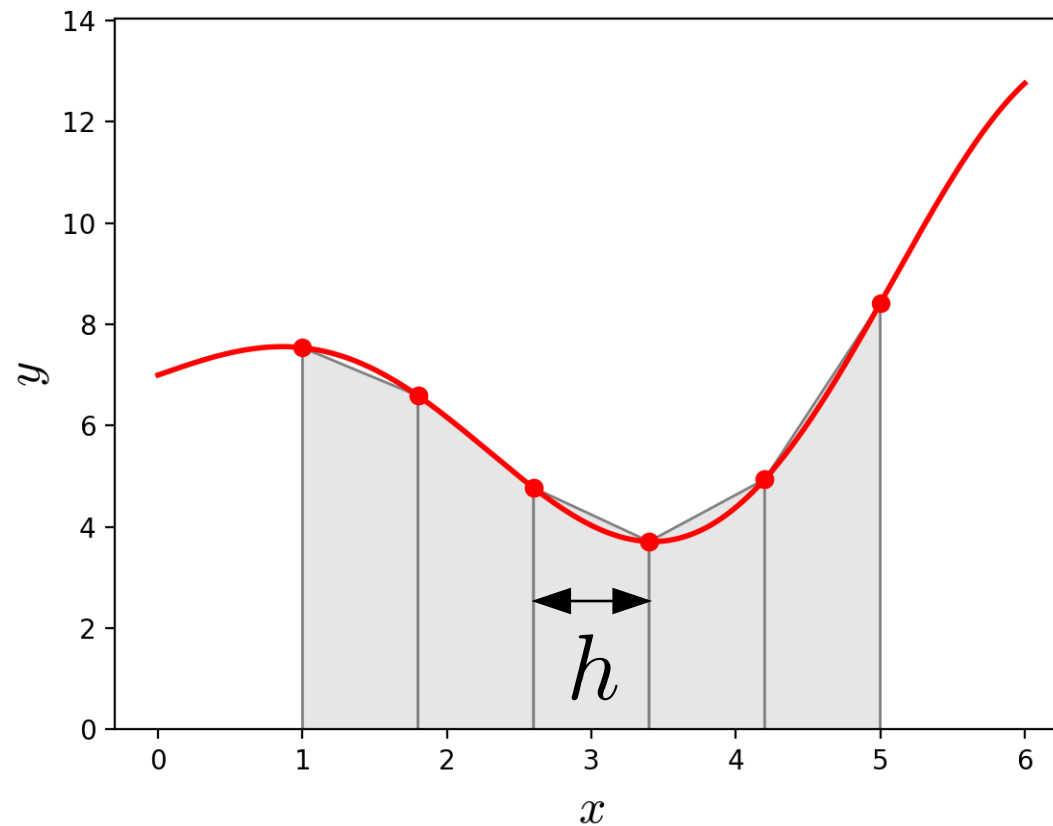

$$N = 5$$

Summierte Trapezregel

$$f(x) = x \cos(x) + 7$$
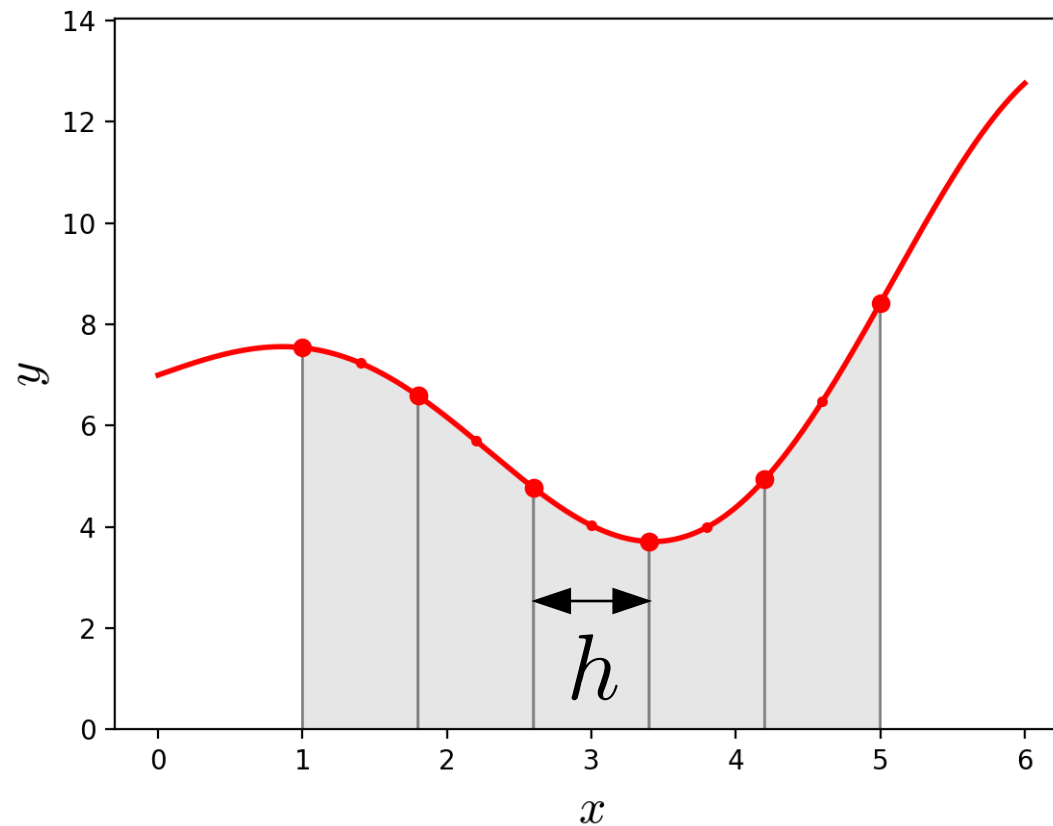

$$N = 5$$

Summierte Simpson-Regel

$$f(x) = x \cos(x) + 7$$

$$N = 5$$
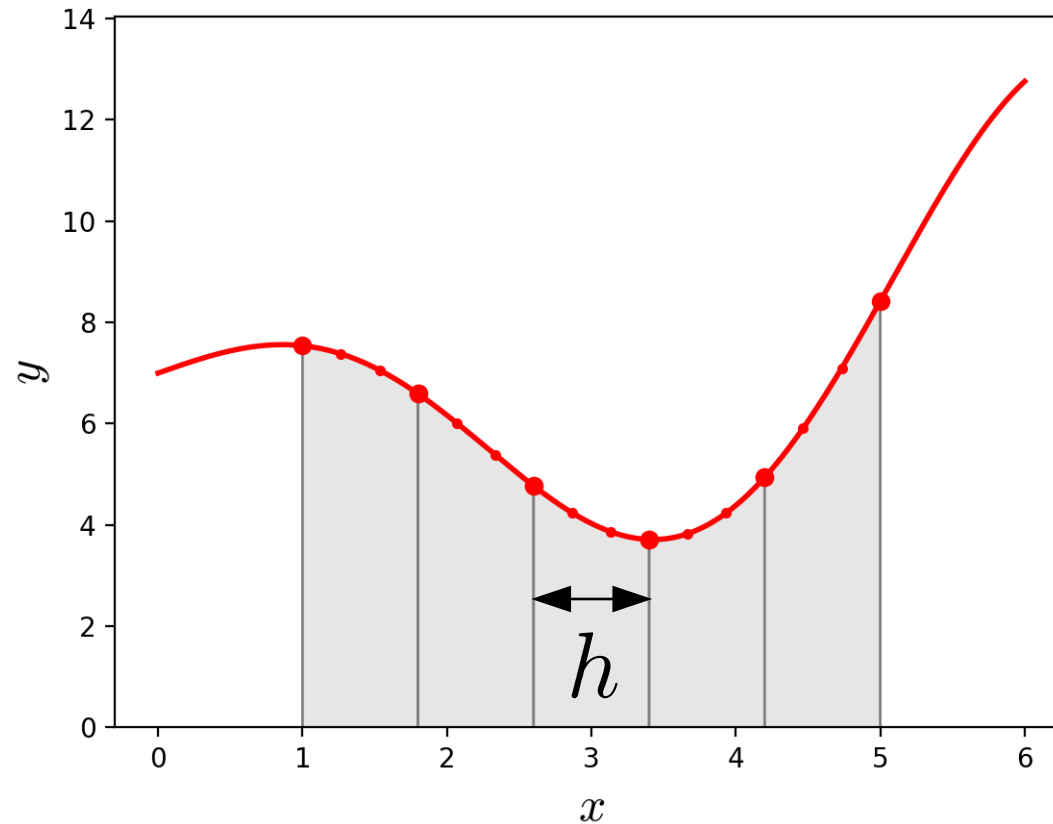
Summierte 3/8-Regel

$$f(x) = x\cos(x) + 7$$

$$N = 5$$
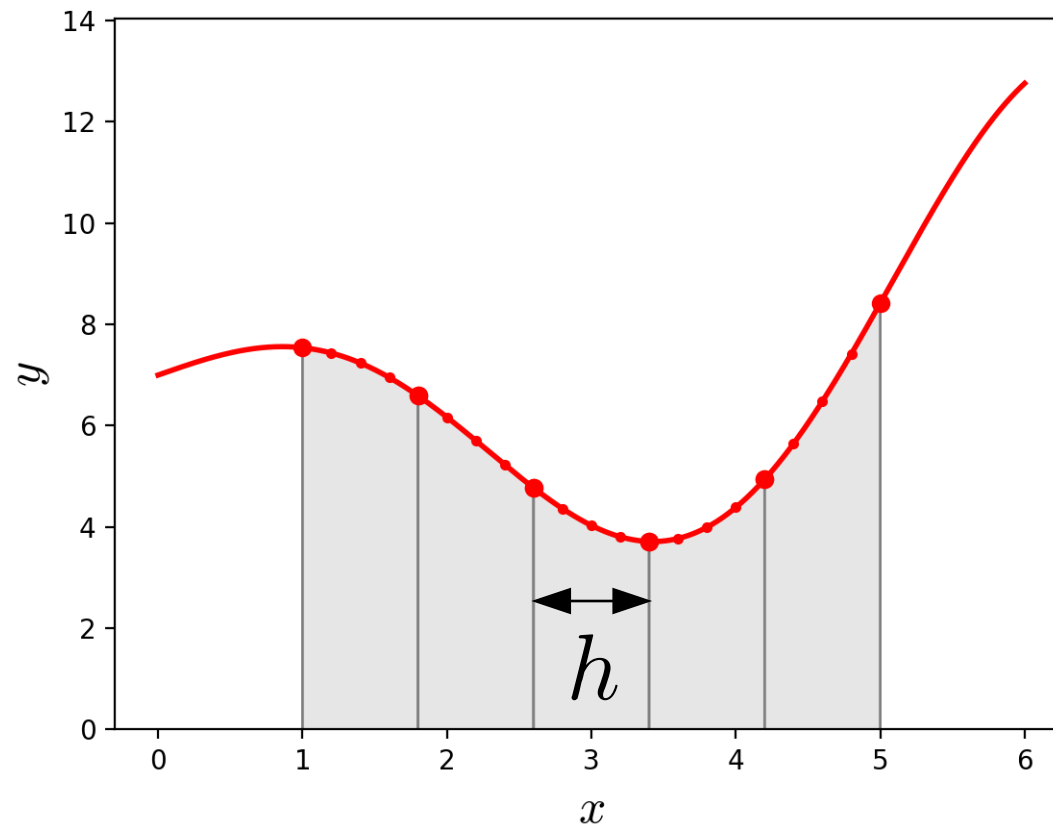
Summierte Milne-Regel

$$f(x) = x\cos(x) + 7$$

$$N = 5$$

Summierte Quadratur-Regeln
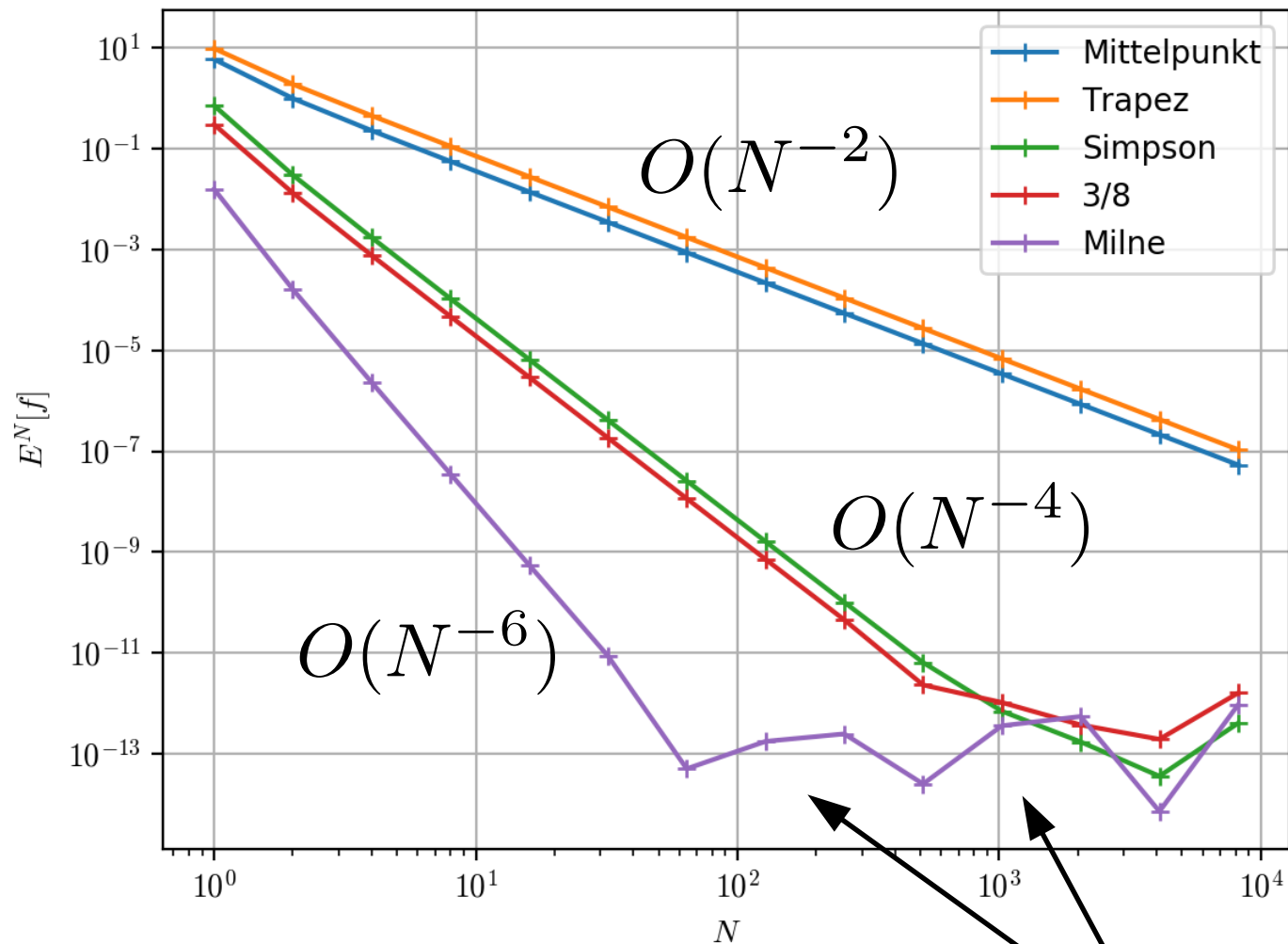
$$f(x) = x \cos(x) + 7$$

N	h	Mittelpunkts $E_0^N[f]$	Trapez $E_1^N[f]$	Simpson $E_2^N[f]$	3/8 $E_3^N[f]$	Milne $E_4^N[f]$
1	1.00E+00	5.99E+00	9.81E+00	7.21E-01	3.05E-01	1.59E-02
2	5.00E-01	1.00E+00	1.91E+00	3.02E-02	1.33E-02	1.62E-04
4	2.50E-01	2.30E-01	4.55E-01	1.74E-03	7.69E-04	2.32E-06
8	1.25E-01	5.64E-02	1.12E-01	1.06E-04	4.72E-05	3.54E-08
16	6.25E-02	1.40E-02	2.80E-02	6.61E-06	2.94E-06	5.50E-10
32	3.12E-02	3.50E-03	7.01E-03	4.13E-07	1.83E-07	8.62E-12
64	1.56E-02	8.76E-04	1.75E-03	2.58E-08	1.15E-08	4.97E-14
128	7.81E-03	2.19E-04	4.38E-04	1.61E-09	7.17E-10	1.74E-13

$$h = \frac{b - a}{N} \qquad \textbf{...}$$

Summierte Quadratur-Regeln

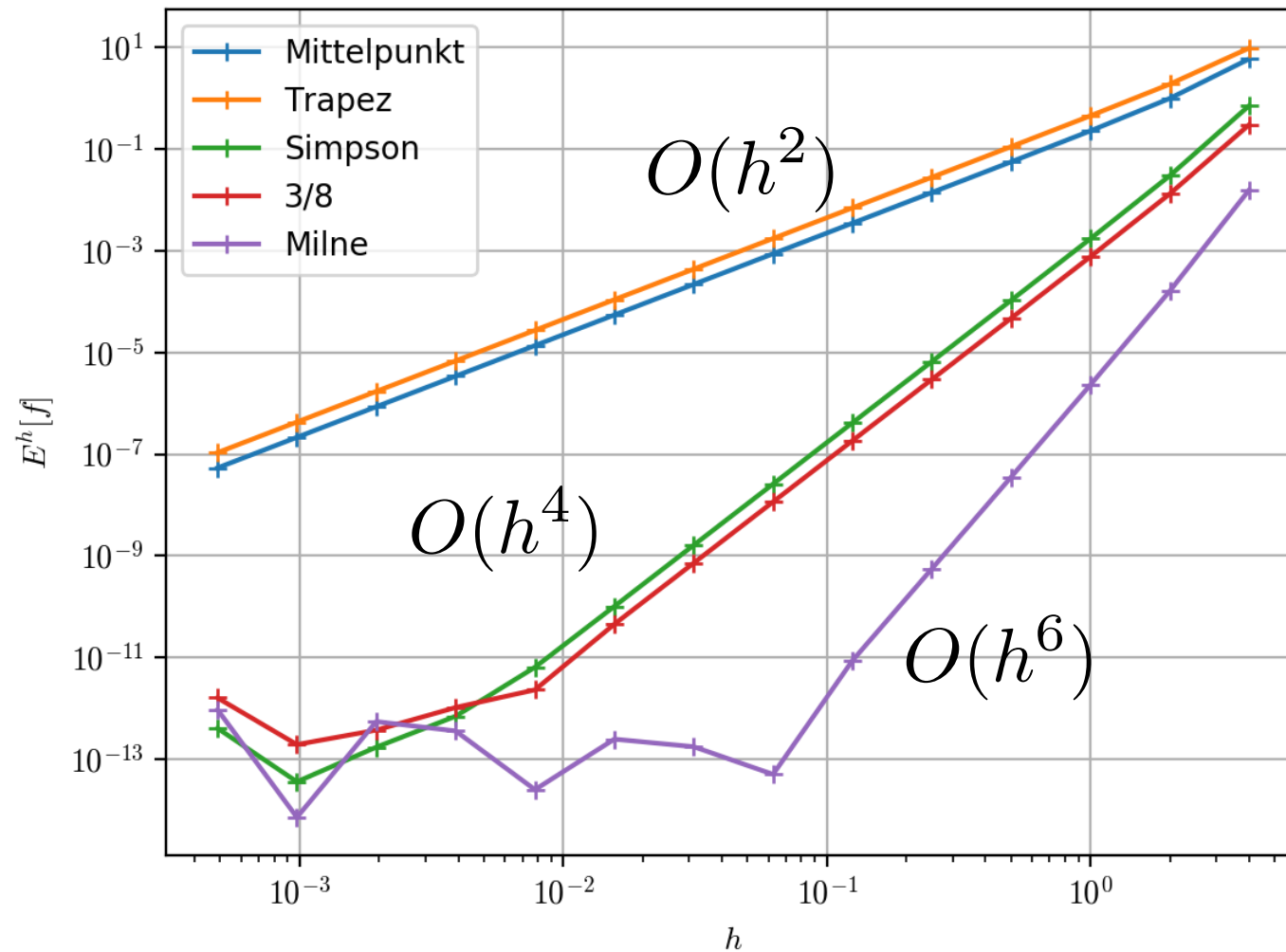$$f(x) = x\cos(x) + 7$$
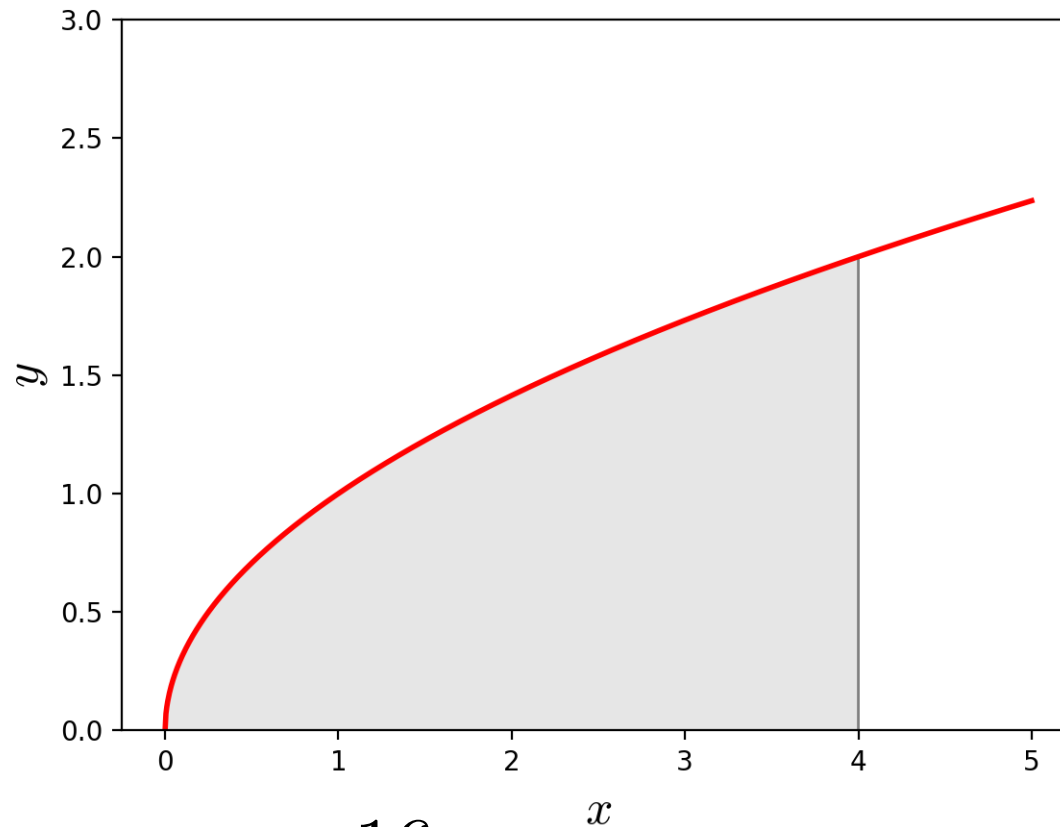
Summierte Quadratur-Regeln

$$f(x) = x\cos(x) + 7$$
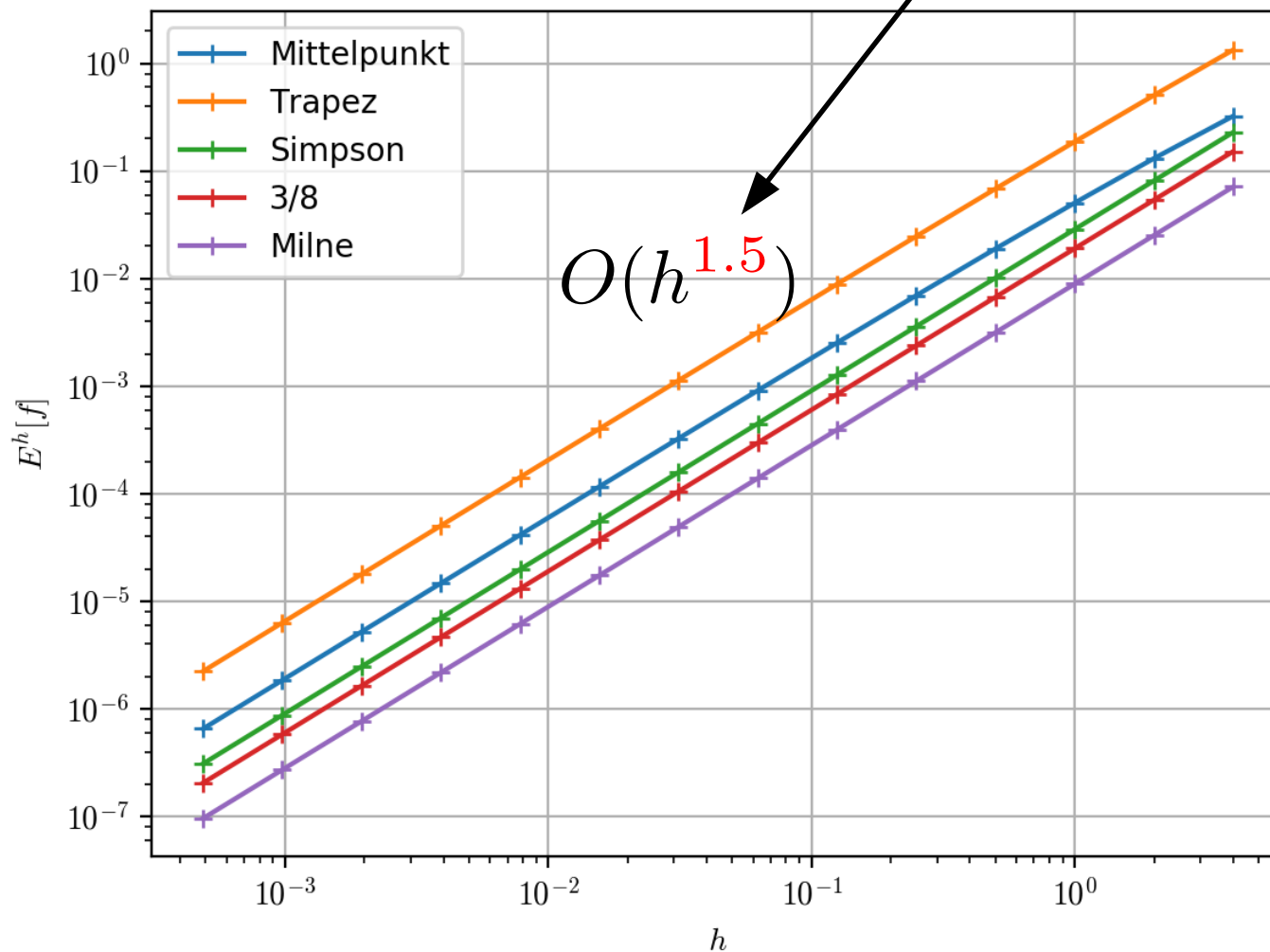
Summierte Quadratur-Regeln

$$f(x) = \sqrt{x}$$

$$I[f] = \int_0^4 f(x)\mathrm{d}x = \frac{16}{3}$$

Summierte Quadratur-Regeln

$$f(x) = \sqrt{x} \qquad \textcolor{red}{?}\ f'(x) = \frac{1}{2\sqrt{x}} \xrightarrow{x \to 0} \infty$$

...Glattheit!

$O(h^{1.5})$

Adaptive Quadratur

$$I[f] = \int_{3/2}^{4} \left(\frac{200}{2x^3 - x^2} \right) \left(5\sin(20/x) \right)^2 \, \mathrm{d}x$$

Adaptive Quadratur

$$I[f] = \int_{3/2}^{4} \left(\frac{200}{2x^3 - x^2} \right) \left(5\sin(20/x) \right)^2 \mathrm{d}x$$

Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x^2} \, \mathrm{d}x$$

Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x^2}\,\mathrm{d}x$$

Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x} \sin(4\pi x)\mathrm{d}x$$

Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x}\sin(4\pi x)\mathrm{d}x$$