

Berechnen wir zunächst die Ableitungen der Lösung

$$\overset{\text{DGL}}{\dot{y}(t)} = f(t, y(t))$$

$$\ddot{y}(t) = \frac{d}{dt} f(t, y(t))$$

$$\overset{\text{Ketten-Regel!}}{=} \frac{\partial f}{\partial t}(t, y(t)) + \frac{\partial f}{\partial y}(t, y(t)) \cdot \dot{y}(t) \quad \text{"} f(t, y(t)) \text{ DGL!}$$

$$= \frac{\partial f}{\partial t}(t, y(t)) + \frac{\partial f}{\partial y}(t, y(t)) \cdot f(t, y(t))$$

$$\ddot{y}(t) = \frac{d}{dt} \left(\frac{\partial f}{\partial t}(t, y(t)) + \frac{\partial f}{\partial y}(t, y(t)) \cdot f(t, y(t)) \right)$$

$$= \frac{\partial^2 f}{\partial t^2}(t, y(t)) + \frac{\partial^2 f}{\partial y \partial t}(t, y(t)) \cdot \dot{y}(t) \quad \text{"} f(t, y(t))$$

$$\overset{\frac{\partial^2 f}{\partial t \partial y} = \frac{\partial^2 f}{\partial y \partial t}}{+} \left(\frac{\partial^2 f}{\partial t \partial y}(t, y(t)) + \frac{\partial^2 f}{\partial y^2}(t, y(t)) \cdot \dot{y}(t) \right) \cdot f(t, y(t)) \quad \text{"} f(t, y(t))$$

$$+ \frac{\partial f}{\partial y}(t, y(t)) \cdot \left(\frac{\partial f}{\partial t}(t, y(t)) + \frac{\partial f}{\partial y}(t, y(t)) \cdot \dot{y}(t) \right) \quad \text{"} f(t, y(t))$$

$$= \frac{\partial^2 f}{\partial t^2}(t, y(t)) + 2 \cdot \frac{\partial^2 f}{\partial t \partial y}(t, y(t)) \cdot f(t, y(t))$$

$$+ \frac{\partial^2 f}{\partial y^2}(t, y(t)) \cdot f(t, y(t))^2 + \frac{\partial f}{\partial t}(t, y(t)) \cdot \frac{\partial f}{\partial y}(t, y(t))$$

$$+ \left(\frac{\partial f}{\partial y}(t, y(t)) \right)^2 \cdot f(t, y(t))$$

$$\vdots$$

$$y^{(k)}(t) = \left(\frac{\partial}{\partial t} + f(t, y(t)) \frac{\partial}{\partial y} \right)^{k-1} f(t, y(t))$$