(18) Verbesserter Euler

$$k_1 = f(t_j, y(t_j))$$

$$k_2 = f(t_j + h/2,\ y(t_j) + h/2\, k_1)$$

$$y_{j+1} = y(t_j) + h \cdot k_2$$

Entwickeln der Verfahrens-Funktion

$$\phi(t_j, y(t_j), h) = f(t_j + \tfrac{h}{2},\ y(t_j) + \tfrac{h}{2} k_1)$$

$$= f(t_j, y(t_j)) \leftarrow \phi(t_j, y(t_j), 0)$$

$$\phi'(t_j, y(t_j), 0) \cdot h \Big\{ + \frac{\partial f}{\partial t}(t_j, y(t_j)) \cdot \frac{h}{2} + \frac{\partial f}{\partial y}(t_j, y(t_j)) \cdot \frac{h}{2} k_1$$

$$\phi''(t_j, y(t_j), 0) \cdot \frac{h^2}{2} \quad \Big\{ + \frac{1}{2} \frac{\partial^2 f}{\partial t^2}(t_j, y(t_j)) \cdot \left(\frac{h}{2}\right)^2 + \frac{\partial^2 f}{\partial t \partial y}(t_j, y(t_j)) \cdot \left(\frac{h}{2}\right)^2 \cdot k_1$$

$$+ \frac{1}{2} \frac{\partial^2 f}{\partial y^2}(t_j, y(t_j)) \cdot \left(\frac{h}{2} k_1\right)^2$$

$$+ \dots$$

Durch Vergleich

$$\phi(t_j, y(t_j), 0) = f(t_j, y(t_j)) = \dot{y}(t_j) \quad \checkmark$$

2. $$\phi'(t_j, y(t_j), 0) = \left(\frac{\partial f}{\partial t}(t_j, y(t_j)) + \frac{\partial f}{\partial y}(t_j, y(t_j)) \cdot f(t_j, y(t_j)) \right)$$

$$= \ddot{y}(t_j) \quad \checkmark$$

3. $$\phi''(t_j, y(t_j), 0) = \dots \quad \times$$