Also $\beta_0 = 1$, $\alpha_0 = 1$, $\alpha_1 = -1$.

(10) BDF2 : $k=2$

Bestimme das Interpolationspolynom durch y_{j+1}, y_j, y_{j-1}:

$$\begin{aligned} p_2(t) = \; & y_{j-1} \cdot \frac{1}{2h^2}(t-t_j)(t-t_{j+1}) \\ & - y_j \cdot \frac{1}{h^2}(t-t_{j-1})(t-t_{j+1}) \\ & + y_{j+1} \cdot \frac{1}{2h^2}(t-t_{j-1})(t-t_j) \end{aligned}$$

Die Ableitung zur Zeit t_{j+1} ist

$$\frac{d}{dt} p_2(t)\Big|_{t=t_{j+1}} = y_{j-1} \cdot \frac{1}{2h} - y_j \cdot \frac{2}{h} + y_{j+1}\frac{3}{2h}$$
$$\approx \dot{y}(t)$$

Und damit

$$\frac{1}{h}\left(\frac{1}{2} y_{j-1} - 2y_j + \frac{3}{2} y_{j+1}\right) = f(t_{j+1}, y_{j+1})$$

Oder

$$\underbrace{y_{j+1}}_{\alpha_0 = 1} \underbrace{- \frac{4}{3}}_{=\alpha_1} y_j + \underbrace{\frac{1}{3}}_{=\alpha_2} y_{j-1} = \underbrace{\frac{2}{3}}_{=\beta_0} h\, f(t_{j+1}, y_{j+1})$$