

Assignment 17

FIXED SUBFIELD

1. Let E/k be a splitting field of $X^n - 1 \in k[X]$ and $\Gamma_n(E)$ the subgroup of E^\times of n -th roots of unity. Show that
 - (a) If $\text{char}(k) = 0$, then $|\Gamma_n(E)| = n$.
 - (b) If $\text{char}(k) = p$, and $n = p^\ell m$ with $p \nmid m$, then $|\Gamma_n(E)| = m$.
2. Let $E = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Recall that $\text{Gal}(\mathbb{Q}(\sqrt{2}, \sqrt{3})/\mathbb{Q}) \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. List all subgroups of $\text{Gal}(\mathbb{Q}(\sqrt{2}, \sqrt{3})/\mathbb{Q})$ and for each subgroup H determine the subfield E^H .
3. Let p be a prime number and let $m \geq 1$. Prove that the field extension $\mathbb{F}_{p^m}/\mathbb{F}_p$ is Galois and calculate its Galois group.

Hint. Frobenius automorphism.