Assignment 21

CONSTRUCTIONS WITH STRAIGHTEDGE AND COMPASS.

1. (a) Express $\cos \frac{\pi}{12}$ in terms of real square roots.

   (b) Prove that an angle α can be trisected if and only if $4X^3 - 3X - \cos \alpha$ is reducible over $\mathbb{Q}(\cos \alpha)$.

2. (Constructible polygons)

   (a) Is the regular 9-gon constructible?

   (b) Deduce that the regular 15-gon is constructible from the fact that the regular 3-gon and the regular 5-gon are.

   (c) More generally, prove that if m and n are coprime and both the m-gon and the n-gon are constructible, then so is the mn-gon.

3. Is it possible to construct a square whose area is that of a given triangle?

4. Show that a root α of $f(X) := X^4 - 4X + 2 \in \mathbb{Q}[X]$ generates a degree 4 extension of $\mathbb{Q}$ but is not constructible.

   Hint: The splitting field of f over $\mathbb{Q}$ has Galois group S_4.