Topology **Exercise Sheet 10**

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We recall that a topological space X is *contractible* if there exists a continuous map $F\colon X \times [0,1] \to X$ and a point $p \in X$ such that for each $x \in X$ we have that $F(x,0) = x$ and $F(x,1) = p$.

Ex.1:

Let X be a convex subset of $\mathbb{R}^n$, for some $n < \infty$. Show that X is contractible.

Ex.2:

Let X be a topological space, and let γ_1, γ_2 be paths in X with the same endpoints (i.e. $\gamma_1(0) = \gamma_2(0)$ and $\gamma_1(1) = \gamma_2(1)$). Show that γ_1 and γ_2 are homotopic if and only if there is a continuous map $d\colon D^2 \to X$, such that $\gamma_1(s) = d((1-s)\pi, 1)$ and $\gamma_2(s) = d((1+s)\pi, 1)$, where the disk is parametrized in polar coordinates.

Ex.3:

Let X be a path connected topological space. Show that $\pi_1(X) = \{1\}$ if and only if for every pair of points x, y of X, there exists only one homotopy class of paths joining them.

Ex.4:

Let X, Y be topological spaces and let $x \in X$ and $y \in Y$ be points.

- Show that there is a map
$$p\colon \pi_1\left(X \times Y, (x,y)\right) \to \pi_1(X,x) \times \pi_1(Y,y).$$
- Show that p is a homomorphism of groups.
- Show that the map p is surjective.
- Show that the map p is injective.