Topology | **Exercise Sheet 11**

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Ex.1:

Recall that X is a contractible space when the identity map on X is null-homotopic, i.e. is homotopic to some constant map in a point. Show that if X is contractible, then X is path connected and $\pi_1(X) \cong \{1\}$.

Ex.2:

Let $I = [0,1]$, and let X be a metric space. Let γ_1, γ_2 be paths in X with the same endpoints. Let $\mathcal{S}$ be the subset of $C(I,X)$ defined as $\{f \in C(I,X) \mid (f(0) = \gamma_1(0), f(1) = \gamma_1(1)\}$. Show that there is a path $\Gamma\colon [0,1] \to C(I,X)$ between γ_1 and γ_2 contained in $\mathcal{S}$, if and only they are homotopic (recall that $C(X,Y) = \{f\colon X \to Y \mid f \text{ is continuous}\}$ with distance defined as $d(f,g) = \sup_{x\in X}\{d_Y(f(x), g(x))\}$).
Hint: you can use Heine-Cantor Theorem.

Ex.3:

Let $p\colon \mathbb{R} \to S^1$ be the map defined as

$$p(t) = (\cos(2\pi t), \sin(2\pi t)).$$

Show that p is a covering map.

Ex.4:

Let $p\colon X \to Y$ be a covering map, and let $F\colon [0,1] \times [0,1] \to Y$ be an homotopy between two paths. For each $y \in Y$, let U_y be an evenly covered neighbourhood of y. Show that there is $n > 0$ such that subdividing $[0,1] \times [0,1]$ is squares of side length $\frac{1}{n}$ we obtained that the image under F of every such sub-square is contained in U_y, for some $y \in Y$.

Ex.5:

Let X, Y, Z be topological spaces, and assume that $p\colon X \to Y$ and $q\colon Y \to Z$ are covering maps. Assume, moreover, that for each $z \in Z$, we have that $q^{-1}(z)$ is a finite set. Show that $q \circ p\colon X \to Z$ is a covering map.