Topology
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Exercise Sheet 7
Due to 10 April

Ex.1:

Let X and Y be topological spaces. Suppose that Y is Hausdorff and that there is a continuous bijection $f\colon X \to Y$. Show that X is Hausdorff.

Ex.2:

Let X be a Hausdorff topological space and let Y be a subset of X. Show that Y is Hausdorff with respect to the induced topology.

Ex.3:

Consider the following two facts:

1. A homeomorphism between locally compact Hausdorff spaces extends to a homeomorphism between the one-point compactifications. In other words, homeomorphic locally compact Hausdorff spaces have homeomorphic one-point compactifications.

2. Given a point $p \in S^n$, the one point compactification of $S^n \setminus \{p\}$ is S^n.

Use those facts to prove that the one-point compactification of $\mathbb{R}$ is homeomorphic to the 1-dimensional sphere $S^1 \subseteq \mathbb{R}^2$.

Can you generalise such homeomorphism to the n-dimensional case (i.e. write down the homeomorphism between $\mathbb{R}^n$ and $S^n \setminus \{p\}$)?

Ex.4:

A map $f\colon X \to Y$ is called *proper* if for every compact $K \subseteq Y$ we have that $f^{-1}(K)$ is compact. Let $f\colon X \to Y$ be a proper, continuous map. Show that f extends to a continuous map $\widehat{f}\colon \widehat{X} \to \widehat{Y}$.

Ex.5:

Given the statement *'Every discrete subset of a compact set is finite'*, argue if it is true or not. If yes, prove it, if no, find a counterexample.

We recall that a subset Y of a topological space is discrete if every point $y \in Y$ is open in the subset topology (note that this implies that every point of Y is open and closed in the subset topology).