Topology **Exercise Sheet 9**

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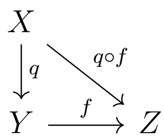
Ex.1:

Let X be a topological space, $q\colon X \to Y$ be the quotient map and let $f\colon Y \to Z$ be any function. Then f is continuous if and only if $f \circ q$ is continuous.



Ex.2:

Let X be a Hausdorff space and let K be a non-open compact in X. Show that the quotient X/K is Hausdorff.

Hint: It can be helpful to write down explicitly how the elements $[x] \in X/K$ look like, distinguishing the case $x \in K$ or not.

Ex.3:

Let X be a topological space, and let Δ be the diagonal of $X \times X$, i.e. the set $\Delta = \{(x, y) \in X \times X \mid x = y\}$. Show that X is Hausdorff if and only in Δ is closed in $X \times X$.

Ex.4:

Let X be Hausdorff, and let $\sim$ be an equivalence relation on X. Let $R = \{(x, y) \in X \times X \mid x \sim y\}$. Suppose that $p\colon X \to X/_\sim$ is open. Show that $X/_\sim$ is Hausdorff if and only if R is closed in $X \times X$.

Ex.5:

Show that there is a quotient map $q\colon (-2, 2) \to [-1, 1]$, but not a quotient map $p\colon [-2, 2] \to (-1, 1)$ (This means that there is a quotient of $(-2, 2)$ that is homeomorphic to $[-1, 1]$).