

Probability and Statistics

Exercise sheet 1

Exercise 1.1 Each of 3 people toss a fair coin. What is the probability of someone being the “odd man out”? This means that 2 of the players obtain the same outcome, while the third gets a different one.

Exercise 1.2 An urn contains 5 red, 5 black and 5 white balls. 3 balls are chosen without replacement at random. What is the probability that they are of exactly two different colors?

Exercise 1.3 3 families with three members each organize themselves randomly into a line, to take a random picture. What is the probability that members of each family appear together in the picture? (i.e., not separated by members from other families.)

Exercise 1.4 In a building with 6 floors (plus the ground floor), an elevator starts with 4 people at the ground floor. What is the probability that these people get off at exactly 2 floors?

Exercise 1.5 Consider an arbitrary sample space Ω and probability measure P . Using induction, prove that for any events $A_1, \dots, A_n \subseteq \Omega$,

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{k=1}^n (-1)^{k+1} \sum_{1 \leq i_1 < \dots < i_k \leq n} P\left(\bigcap_{j=1}^k A_{i_j}\right).$$