Probability and Statistics

Exercise sheet 6

Exercise 6.1 (Binomial disguised)

Suppose that each day the price of a stock moves up 12.5 cents with probability $\frac{1}{3}$ and down 12.5 cents with probability $\frac{2}{3}$.

If the movements of the stock from one day to another are independent, what is the probability that after 10 days the stock has its original price?

Exercise 6.2 (The coffee is on me if...)

Barbara, Christa and Cecilia are going to have coffee at the local coffee shop. They will each toss a fair coin, and if one of them comes out as the "odd woman out" (the one with the different outcome), then she pays for all three. They keep tossing until an odd woman is found.

What is the probability that a decision will be reached with 2 rounds of tosses?

Optional: Can you generalize that to n people, coins with probability p of obtaining heads and the question being if a decision is reached within k rounds of tosses?

Exercise 6.3 (Domination of the minority)

In a small town of Alaska, there are 60 Republicans and 40 Democrats. 10 are selected at random for a council. What is the probability that there will be more Democrats than Republicans?

Exercise 6.4 A Poisson process with rate λ per time unit is a random process that has to satisfy the following properties:

1. The number of arrivals in every fixed interval of time of length t has the Poisson distribution with rate λt.
2. The number of arrivals in every collection of disjoint time intervals are independent.

Suppose that the arrival time of customers to some store A is a Poisson process with rate 1 per hour.

(a) What is the probability that the first customer arrives after time t?

(b) What is the probability that "strictly more than 2 customers come during the first hour"?

(c) Fix a time $t > 0$. What is the probability that "a customer comes exactly at time t"? Does this mean that nobody comes at any time? How should we interpret what seems to be a contradiction?

(d) Suppose that the arrival time of customers to another store B is a Poisson process with rate μ per hour, and it is independent of the Poisson process of the arrival time of customers of store A. Now, we assume again that A has a general rate of λ per hour, instead of 1.

How can you describe the arrival times of clients to either of the two stores (not caring which store they go to)?

Exercise 6.5 (optional)

(a) From the set $\{1, 2, ..., N\}$, with $N \geq 2$ an integer, draw i.i.d random variables $X_1, ..., X_n$ with uniform distribution.

What is

$$P\left(\max_{1\leq i\leq n}(X_i) = k\right)$$

for some fixed $k \in \{1, ..., N\}$?

Hint: Recall that for any random variable Y,

$$P(Y = y) = F(y) - F(y-) = P(Y \leq y) - P(Y < y).$$

(b) What if now $X_1, ..., X_n$ are drawn randomly from $\{1, 2, ..., N\}$ without replacement?