Probability and Statistics

Exercise sheet 9

Exercise 9.1 (A gambling example).

It costs 1 dollar to play a certain slot machine in Las Vegas. The machine is set by the house to pay 2 dollars with probability 0.45 and to pay nothing with probability 0.55.

Let X_i be the house's net winnings on the i^{th} play of the machine.

Let $S_n := \sum_{i=1}^{n} X_i$ be the house's winnings after n plays of the machine. Assuming that successive plays are independent, find:

(a) $E(S_n)$;

(b) $\mathrm{var}(S_n)$;

(c) the approximate probability that after $10,000$ rounds of the machine, the house's winnings are between 800 and $1,100$ dollars ($800 \le S_{10,000} \le 1,100$).

Exercise 9.2 Suppose that the cost of a textbook at the college level is on average 50 francs, with a standard deviation of 7 francs.

In a four year bachelor's program, a student will need to buy 25 textbooks, the prices of which are assumed to be independent and identically distributed. Find an approximation to the probability that the student will have to spend more than 1300 francs on textbooks.

Exercise 9.3 (Comparing a Poisson approximation and a normal approximation).

Suppose 1.5% of residents of a town never read a newspaper. We draw a random sample of 50 people, and we want to determine the probability that at least 1 resident in the sample never reads a newspaper.

For that probability, compute:

(a) the exact value;

(b) a Poisson approximation;

(c) a normal approximation.

Exercise 9.4 Consider the joint pmf

$$p(x,y) = \begin{cases} cxy, & 1 \le x \le 3, 1 \le y \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the normalising constant c.

(b) Are X and Y independent? Why?

(c) Find $E(X)$, $E(Y)$ and $E(XY)$.

Exercise 9.5 Let (X, Y) be a discrete random pair.

(a) Show that

$$\mathrm{var}(X) = \mathrm{var}_Y(E(X \mid Y = y)) + E_Y(\mathrm{var}(X \mid Y = y)).$$

(b) Suppose that $Y \sim \mathrm{Poi}(\lambda)$ for some $\lambda \in (0, +\infty)$ and $X \mid Y = y \sim \mathrm{U}(\{0, 1, ..., y\})$. Find $E(X)$ and var(X).

(c) Now, suppose that X and Y are $\overset{\text{iid}}{\sim}$ Geo(p) for some $p \in (0, 1)$. What is $P(X \geq Y)$? And $P(X > Y)$?