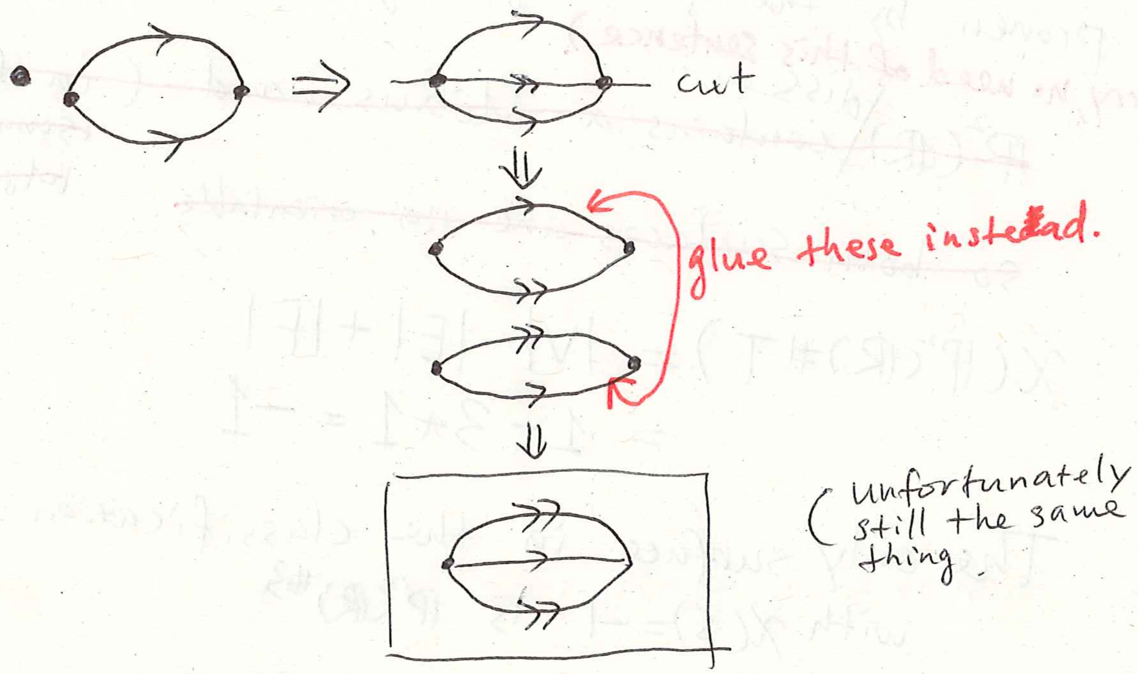
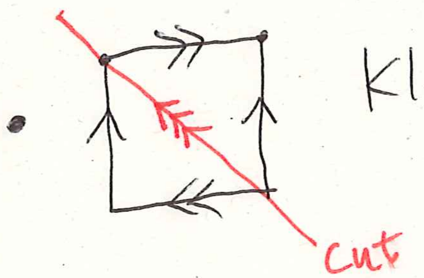
Extra: Cut and paste

Given an identification polygon, there is an operation to create a different polygon representing the same surface.

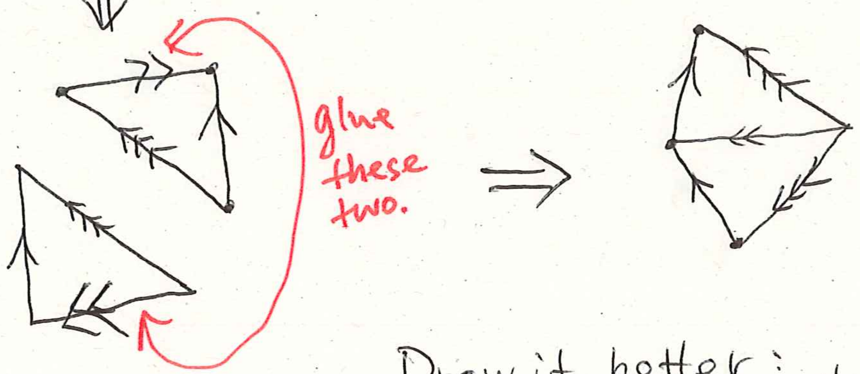
Example:



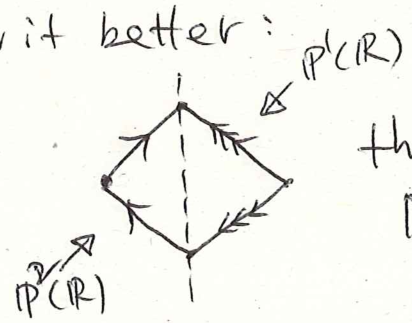
(unfortunately still the same thing.)



Klein bottle



Draw it better:



this is in fact $\mathbb{P}^2(\mathbb{R}) \# \mathbb{P}^2(\mathbb{R})$

• Exercise

$$\mathbb{P}^2(\mathbb{R}) \# T \cong \mathbb{P}^2(\mathbb{R}) \# \mathbb{P}^2(\mathbb{R}) \# \mathbb{P}^2(\mathbb{R})$$

Assuming the classification theorem, this can be proven by the following argument:

(sorry, no need of this sentence).

~~$\mathbb{P}^2(\mathbb{R}) \setminus$ disc contains a Möbius band (in fact it IS isomorphic to Möbius band!)~~

~~so both surfaces are non-orientable~~.

$$\chi(\mathbb{P}^2(\mathbb{R}) \# T) = |V| - |E| + |F|$$
$$= 1 - 3 + 1 = -1$$

The only surface in the classification list with $\chi(S) = -1$ is $\mathbb{P}^2(\mathbb{R})^{\#3}$