Extra: Degree-genus formula

~~Ex~~ $X = V(y^{d-1}z - (x-a_1 z)\cdots(x-a_d z) = 0)$

<u>HW:</u> Similar to the elliptic curve example,

$$\begin{array}{ccc} X & \longrightarrow & \mathbb{P}^1 \\ [x:y:z] & \longmapsto & [x:z] \quad \text{if } (x,z)\neq 0 \\ [0:1:0] & \longmapsto & [0:1] \end{array}$$

is a hol. map.

What are the ramificat'n points?
What are their rami. indices?

What I want to do:

<u>Lem</u>: $X_1 = V(F)$, $X_2 = V(G)$ are ~~sub~~ smooth projective curves in $\mathbb{P}^2$ and $\deg(F) = \deg(G) = d$.
~~If X_1, X_2 are smooth,~~ The ir ~~genus~~ genera are the same.

Sketch of pf: "Universal family":

$$\mathcal{X}_d = V\Big(\sum_{a_1+a_2+a_3=d} A_{a_1a_2a_3}\, x^{a_1} y^{a_2} z^{a_3}\Big) \subset \mathbb{P}^N \times \mathbb{P}^2$$

$$[\cdots A_{a_1a_2a_3}\cdots] \times [x:y:z]$$

$$\Big(N = \binom{d+3}{2}\Big)$$

<u>Check</u>: $\mathcal{X}_d$ is a smooth submfd

Now X_1, X_2 are fibers of the projection π.

$$\begin{array}{ccccc} X_1 & & \mathcal{X}_d & & X_2 \\ \downarrow & & \downarrow \pi & & \downarrow \\ \{F\} & \in & \mathbb{P}^N & \ni & \{G\} \end{array}$$

~~Check~~

~~Fact~~

homog. of deg. d

$Z_{sing} = \{ f \mid V(f) \text{ is singular} \} \subset \mathbb{P}^N$ is defined by polynomials

Fact: $\mathbb{P}^N - Z_{sing}$ is path-connected

(e.g., one can show that Z_{sing} is a ~~complex~~ CW-cplx of real dimension $\leq N-2$)

By Ehresmann theorem,

$$\begin{array}{c} \mathcal{X}_d - \pi^{-1}(Z_{sing}) \\ \downarrow \\ \mathbb{P}^N - Z_{sing} \end{array} \quad \text{is a smooth fibration}$$

Fibers are homeomorphic to each other. $\square$