

Extra: Degree-genus formula

~~HW~~ $X = V(y^{d-1}z - (x-a_1z)\dots(x-a_dz) = 0)$

HW: Similar to the elliptic curve example,

$$X \longrightarrow \mathbb{P}^1$$

is a hol. map.

$$[x:y:z] \longmapsto [x:z] \text{ if } (x,z) \neq 0$$

$$[0:1:0] \longmapsto [0:1]$$

What are the ramification points?
 What are their ramification indices?

What I want to do:

Lemma: $X_1 = V(F)$, $X_2 = V(G)$ are ~~sub~~ ^{smooth} projective curves in \mathbb{P}^2 and $\deg(F) = \deg(G) = d$.

~~If X_1, X_2 are smooth, then their genera are~~
 the same.

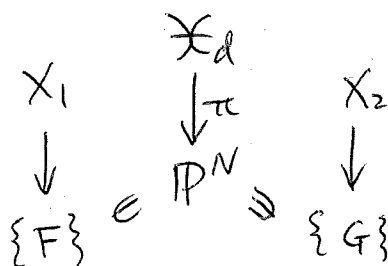
Sketch of Pf: "Universal family":

$$\mathcal{X}_d = V\left(\sum_{a_1+a_2+a_3=d} A_{a_1 a_2 a_3} x^{a_1} y^{a_2} z^{a_3}\right) \subset \mathbb{P}^N \times \mathbb{P}^2$$

$[A_{a_1 a_2 a_3}] \times [x:y:z]$
 $(N = \binom{d+3}{2})$

Check: \mathcal{X}_d is a smooth submfld

Now X_1, X_2 are fibers of the projection π .



~~check~~

~~Fact~~

homog. of deg. d

$Z_{\text{sing}} = \{ f \mid V(f) \text{ is singular} \} \subset \mathbb{P}^N$ is defined by polynomials

Fact: $\mathbb{P}^N - Z_{\text{sing}}$ is path-connected

(e.g., one can show that Z_{sing} is a ~~complex~~ CW-cplx of ~~real~~ dimension $\leq N-2$)

By Ehresmann theorem,

$$\mathbb{C}^d - \pi^{-1}(Z_{\text{sing}})$$



is a smooth fibration

$$\mathbb{P}^N - Z_{\text{sing}}$$

Fibers are homeomorphic to each other. \square