Sample problems

(The exam problems will not be restricted to the questions shown in this page. This is merely a sample collection. I also recommend to finish as many exercises as possible from the textbook. The exam is a 20min oral exam. The preparation is completely up to yourself. I will be around after July 18th and please let me know if you need any help.)

0.1 Concepts

1.) The definition of $\mathbb{P}^n(\mathbb{C})$.
2.) Transition functions of $\mathbb{P}^n(\mathbb{C})$.
3.) What is a complex torus? What is its universal cover? Also describe the covering map.
4.) What is the definition (or criterion) of smoothness for an affine algebraic curve? What is the criterion of smoothness for a projective algebraic curve?
5.) Prove Riemann-Hurwitz formula.
...

0.2 Examples

1.) Recognize typical identification polygons (sphere, torus, real projective space, $T^{\# g}$, etc.).
2.) Compute the Euler number of a surface. Either an identification polygon is given or the name is given.
3.) Prove that the Klein bottle is homeomorphic to $\mathbb{P}^2(\mathbb{R})\#\mathbb{P}^2(\mathbb{R})$.
4.) What is the affine algebraic curve $V\{x^2+xy+y^2=1\}$ in $\mathbb{C}^2$ homeomorphic to, sphere, plane or punctured plane? Why?
5.) Give an example of a holomorphic map from $\mathbb{P}^1(\mathbb{C})$ to $\mathbb{P}^1(\mathbb{C})$.
6.) Does there exist a holomorphic map from a complex torus to $\mathbb{P}^1(\mathbb{C})$? Does there exist a holomorphic map from $\mathbb{P}^1(\mathbb{C})$ to a complex torus? Name an example or prove its nonexistence.
7.) What are the ramification points, ramification indices, and branch points for the following maps:

1. $f : \mathbb{P}^1 \to \mathbb{P}^1$, $f([x : y]) = [x^d : y^d]$.
2. $f : \mathbb{P}^1 \to \mathbb{P}^1$, $f([x : y]) = [x^2 + y^2 : xy]$
3. $f : E \to \mathbb{P}^1$, $f([x : y : z]) = [x : z]$ for $z \neq 0$ and $f([0 : 1 : 0]) = [1 : 0]$, where $E = V(\{y^2 - x(x-z)(x+z)\}) \subset \mathbb{P}^2$.

8.) What are

1. $H_{0 \xrightarrow{d} 0}((d),(d))$;
2. $H_{1 \xrightarrow{2} 1}()$;
3. $H_{h \xrightarrow{2} 0}((2)^{2g+2})$;
4. $H_{0 \xrightarrow{3} 0}((3),(2,1)^2)$?

9.) What's the monodromy representation of a genus g hyperelliptic cover.
10.) Given a monodromy representation, draw the construction of ramified cover
(e.g., $Y = \mathbb{P}^1$, $B = \{b_1, b_2, b_3\}$, $\Psi : \pi_1(Y - B) \to S_3$, $\Psi(\rho_1) = (123)$, $\Psi(\rho_2) = (13)$, $\Psi(\rho_3) = (12)$ where ρ_i is a small loop around b_i).