Exercise sheet 1: Informal introduction and outlook

1. Prove that there are no knots with crossing number one or two.
2. Prove that the only knots with crossing number three are the trefoil and its mirror image (you do not have to prove that they are different knots).
3. Prove using a piece of string or otherwise that the figure-eight knot is amphichiral.
4. Prove that a knot diagram of the mirror image of a knot can be obtained by replacing all over crossings by under crossings and vice versa of a knot diagram of the original knot.
5. Let K_1 and K_2 be two oriented knots. Prove that the connected sum $K_1 \# K_2$ is well-defined.
6. Prove the unknot is an alternating knot.
7. If one wants to draw a knot at random, the easiest method is to draw in pencil a random immersion of a circle in the plane which intersects itself only in transverse double points, and then rub out pairs of little arcs near each double point to show which arc goes over at that point. Show that by choosing the crossings in a particular way you can always produce a diagram of the unknot.
8. (Not so easy!) A particular sensible way of choosing the crossings is to start from some point on the curve and to trace it, imposing alternation on the crossings. Why does this alternation procedure actually work?

   Hint: Alternating is equivalent (proof?) to a chess-board coloring of the regions of the knot diagram.

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