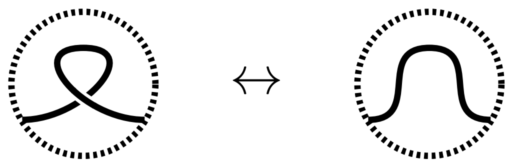
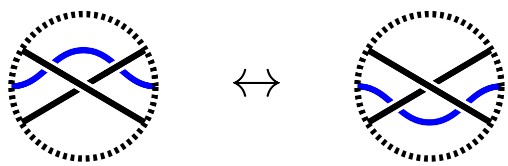
Exercise sheet 3: Reidemeister moves and some simple knot invariants

1. Show that the following variations of the Reidemeister moves can be achieved using $R1$, $R2$ and $R3$ only.

   (a) $\widetilde{R1}$

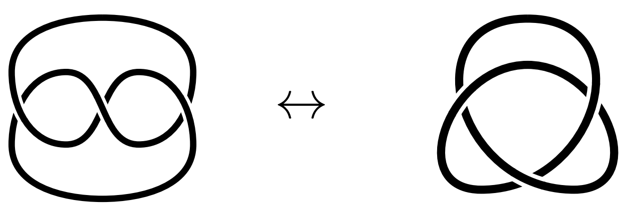
   

   (b) $\widetilde{R3}$

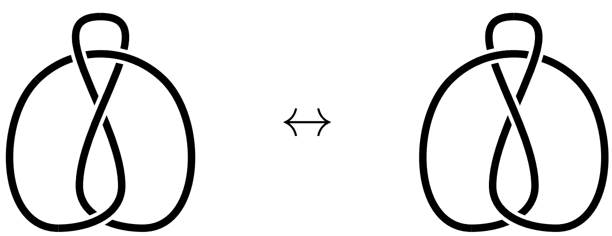
   

2. Write down a sequence of Reidemeister moves to show that the following diagrams are equivalent.

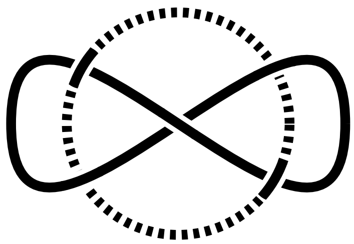
   (a) Two variations of the trefoil

   (b) The figure-eight knot and its mirror image

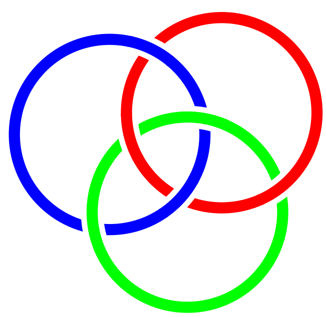
(c) Interchange the components of the Whitehead link



3. Prove that the unknotting number $u(K)$ is finite.

4. (a) Prove that for any oriented link L the linking number $lk(L)$ is an integer.

   (b) Choose an orientation for the following link, the *Borromean rings*, and compute its linking number. How does the linking number change if you reverse one, two or all three of the components?



5. Prove that $c(K)$ and $u(K)$ are subadditive invariants i.e.

$$c(K_1 \# K_2) \leq c(K_1) + c(K_2)$$

and likewise for $u(K)$.

Remark: It is conjectured that both these invariants are in fact additive, i.e. that equality holds above, but no one was able to prove that so far.

6. Prove that the linking number $lk(L)$ is invariant under Reidemeister move 3.

7. Draw a picture to show that there exist oriented 2-component links with linking number n, for any integer n.

8. Prove that if the orientation on one component of a 2-component oriented link L is reversed then its linking number is negated. Moreover, determine the linking number of the mirror image of L.

Due Date: 11.03.2019