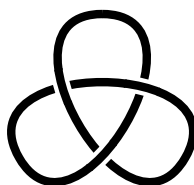
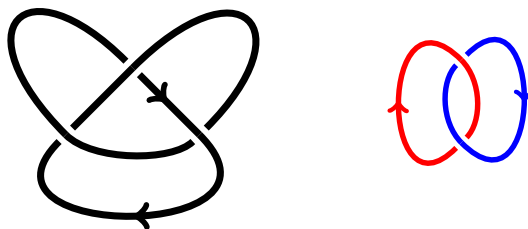


## Exercise sheet 5: The Jones Polynomial I

1. Prove that the Kauffman bracket is invariant under Reidemeister move three.
2. Calculate the bracket polynomial of the usual projection of the trivial link with  $n$  components.
3. Determine the bracket polynomial for the following projection of the trefoil.



4. (a) Show the writhe of a link projection is invariant under Reidemeister moves two and three.  
 (b) Prove the writhe of a link projection has the same parity as the number of crossings  $c$  in this projection.
5. Compute the  $X$ -polynomial of the following two oriented links. What happens to  $X(L)$  if we change the orientation on one or on both components of the links?



6. Determine the Jones polynomial of the trefoil – you can use the result of question 5 – what happens if you reverse your chosen orientation?
7. Let  $c$  be the actual number of crossings in a given diagram.
  - (a) Prove that  $-c \leq \sum s \leq c$ .

- (b) Prove that  $\sum s$  has the same parity as  $c$ .
8. Show that although the Jones polynomial is defined for oriented links, it is actually an invariant for knots (i.e. the orientation is not needed).

**Due Date: 25.03.2019**