Exercise sheet 6: The Jones Polynomial II

1. Prove the Kauffman bracket is well-defined, i.e. show that the explicit state-sum formula equals the Kauffman bracket.

2. Compute the Jones polynomial of the figure-eight knot in two ways: first do it by the Kauffman bracket definition and then using the Jones skein relation - you should get the same result and remember the figure-eight knot is amphichiral.

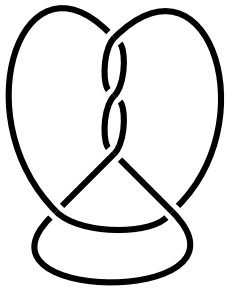
3. (a) Determine the Jones polynomial of the positive Hopf link (the one with linking number $+1$) using the Jones skein relation and show that it is not equivalent to its mirror image.

   (b) Calculate the Jones polynomial of the right-handed trefoil using the Jones skein relation and prove (finally) that it is not equivalent to its mirror image.

4. (a) Prove that $V(K_1 \# K_2) = V(K_1) \cdot V(K_2)$. (Hint: first give a formula for the Kauffman bracket of a connected sum).

   (b) Prove that $V(K_1 \sqcup K_2) = (-t^{\frac{1}{2}} - t^{-\frac{1}{2}}) V(K_1) \cdot V(K_2)$, where $\sqcup$ denotes the disjoint union.

5. Use the axioms to calculate the Jones polynomial of the knot 5_2 shown below – you may use any results you have obtained so far – is this knot amphichiral?



The result should be $V(5_2) = -t^{-6} + t^{-5} - t^{-4} + 2t^{-3} - t^{-2} + t^{-1}$

6. (**Unsolved Question**) Does the Jones polynomial distinguish every knot from the unknot? That is to say, is there a nontrivial knot with Jones polynomial 1?

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