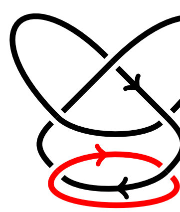
Exercise sheet 8: More about polynomial invariants

1. Calculate the HOMFLY-polynomial of the Hopf-link where both rings carry opposite orientations.
2. Determine the HOMFLY-polynomial of the trefoil.
3. Show that the HOMFLY-polynomial is a knot invariant (i.e. does not depend on the orientation of the knot).
4. Prove the following properties of the HOMFLY-polynomial:
   (a) $P(L \sqcup O) = -(l + l^{-1})m^{-1}P(L)$
   (b) $P(L_1 \sqcup L_2) = -(l + l^{-1})m^{-1}P(L_1)P(L_2)$
   (c) $P(L_1 \# L_2) = P(L_1)P(L_2)$
5. Use the property for the connected sum of the links L_1 given by

   and $L_2 = 4_1$, i.e. the figure-eight knot, to show that the HOMFLY-polynomial is not a complete invariant for oriented links.
6. Explain what it means that the HOMFLY-polynomial determines the Jones-polynomial by setting $l = it^{-1}$ and $m = i(t^{-\frac{1}{2}} - t^{\frac{1}{2}})$.
7. Prove the last part of the proposition about the Conway-polynomial $\nabla_L(z)$ which states that: If $\#L_+ = \#L_- = 1$ then
   $$a_2(L_+) - a_2(L_-) = \mathrm{lk}(L_0)$$
8. Determine $\nabla_{3_1}(z)$ and $\nabla_{\overline{3_1}}(z)$ – what do you observe?

Due Date: 29.04.19