Exercise sheet 9: Crashcourse on surfaces I

1. (a) Show using an explicit map that the crosscap is homeomorphic to the Möbius strip.

   (b) Show that the Klein bottle is homeomorphic to the union of two copies of a Möbius strip joined (by a homeomorphism) along their boundaries.

2. (a) Show that a disc with a twisted handle attached is homeomorphic to a disc with two crosscaps attached.

   (b) Show that a disc with a crosscap and a handle attached is homeomorphic to a disc with three crosscaps attached.

3. (This exercise gives an alternative way of looking at the addition of a handle or twisted handle).

   Let E be the disc of radius 10 in $\mathbb{C}$ minus the open unit discs centered at $z = \pm 5$. Let X be the space $E \cup (S^1 \times I)$, where the cylinder is attached via $(e^{i\theta}, 0) \sim -5 + e^{i\theta}$ and $(e^{i\theta}, 1) \sim 5 + e^{-i\theta}$. Let Y be the identification with $-5 + e^{i\theta} \sim 5 + e^{-i\theta}$.

   (a) Prove explicitly that X, which is the disc with a handle added is homemorphic to Y.

   (b) What happens when the $e^{-i\theta}$'s are replaced with $e^{i\theta}$'s

4. Let F' be the space obtained by cutting a surface F along a closed curve C. Prove the following lemma: There is a continuous "regluing" map $p : F' \to F$. The boundary of F' is $\partial F' = p^{-1}(C) \sqcup \partial F$, and the new part $p^{-1}(C)$ consists of either one or two circles.

5. (a) Show that cutting along a separating curve increases the number of components of a surface by 1.

   (b) Show that cutting along a one-sided curve cannot separate a surface.

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