

Proposition 8.7. *For an oriented link L with $\#L$ components, the Conway polynomial has the following properties.*

- (i) *If L is a split link, then $\nabla_L(z) = 0$.*
- (ii) *$a_i(L) = 0$ for $i \equiv \#L \pmod{2}$ and also for $i < \#L - 1$.*
- (iii) *If L is a knot, so $\#L = 1$, then $a_0(L) = 1$.*
- (iv) *If $\#L = 2$, then $a_1(L) = \text{lk}(L)$, where $\text{lk}(L)$ is the linking number of the two components of L .*
- (v) *If L_+ , L_- and L_0 are related in the manner of Figure 3.2 and $\#L_+ = \#L_- = 1$, then $a_2(L_+) - a_2(L_-) = \text{lk}(L_0)$.*

PROOF. (i) This follows from the stronger Proposition 6.14. However, it also follows at once by applying the skein formula to links L_+ , L_- and L_0 shown in Figure 8.2. As L_+ and L_- are here the same link, $\nabla_{L_0}(z) = 0$.

(ii) This follows by induction on the number of crossings in a diagram and the number of crossing changes needed to make it a diagram of the trivial link of unknots.

(iii) When $z = 0$, the skein formula becomes $\nabla_{L_+}(0) = \nabla_{L_-}(0)$, so any crossings can be changed without altering $\nabla_L(0)$. Of course, $a_0(\text{unknot}) = 1$.

(iv) Suppose the skein formula is considering a crossing between the two components of L . Using (iii), consideration of the coefficient of z shows that $a_1(L_+) - a_1(L_-) = 1$. But $\text{lk}(L_+) - \text{lk}(L_-) = 1$, so the result follows by using (i) and considering a collection of crossing changes that yield a split link.

(v) This follows at once from (iv). □