Applied Stochastic Processes

Exercise sheet 6

Exercise 6.1 Show that a renewal process with renewal function $M(t) = ct$, $t \geq 0$ for some constant $c > 0$ is a Poisson process.

Hint: *The Laplace transform determines the distribution.*

Exercise 6.2 Vehicles of random lengths arrive at a gate. Let L_k denote the length of the k-th vehicle. We assume that the random variables L_k are i.i.d. with distribution $3 + \text{Geometric}(1/2)$. The first vehicle that arrives parks directly at the gate. The vehicles arriving afterwards queue behind, leaving a random distance to the vehicle parked in front of themselves. We assume that these distances are independent and uniformly distributed on $[0, 1]$.

(a) For $x \geq 0$, let N_x denote the number of vehicles parked at distance at most x from the gate. Compute $\lim_{x \to \infty} N_x/x$.

(b) Suppose that the k-th vehicle is carrying D_k people, where the random variable D_k is distributed as $1 + \text{Binomial}(2L_k, 1/2)$. For $x \geq 0$ let $\widetilde{N}_x$ denote the number of people inside the vehicles parked at distance at most x from the gate. Estimate $\widetilde{N}_x$ for x large enough.

Exercise 6.3 Central Limit Theorem for Renewal Processes

If $(N_t)_{t \geq 0}$ is a renewal process with inter-arrival times T_i, $i \geq 1$, not a.s. constant and such that $E[T_1^2] < \infty$, show that when $t \to \infty$,

$$Z_t := \frac{N_t - t/\mu}{\sigma(t/\mu^3)^{\frac{1}{2}}}$$

converges in law to the standard normal distribution, where $\mu = E[T_1]$ and $\sigma^2 = \text{Var}(T_1) > 0$.

Hint: *Let $S_n := T_1 + ... + T_n$, then by the central limit theorem*

$$\lim_{n \to \infty} P[(S_n - n\mu)/\sigma\sqrt{n} \leq x] = \Phi(x)$$

uniformly in $x \in \mathbb{R}$, where Φ denotes the distribution function of the standard normal distribution.

Submission deadline: 13:15, Apr. 4.

Location: During exercise class or in the tray outside of HG E 65.

Class assignment:

Students	Time & Date	Room	Assistant
A-K	Thu 09-10	HG D 7.2	Maximilian Nitzschner
L-Z	Thu 12-13	HG D 7.2	Daniel Contreras

Office hours (Präsenz): Mon. and Thu., 12:00-13:00 in HG G 32.6.

Exercise sheets and further information are also available on:
http://metaphor.ethz.ch/x/2019/fs/401-3602-00L/