Applied Stochastic Processes

Exercise sheet 8

Exercise 8.1  A die is rolled repeatedly. Which of the following stochastic processes \((X_n)_{n \in \mathbb{N}}\) are Markov chains? For those that are, determine the transition probability and in (b), additionally, the \(n\)-step transition probability.

(a) Let \(X_n\) denote the number of rolls at time \(n\) since the most recent six.

(b) Let \(X_n\) denote the largest number that has come up in the first \(n\) rolls.

(c) Let \(X_n\) denote the larger number of those that came up in the rolls number \(n - 1\) and \(n\) (the last two rolls), and we consider \((X_n)_{n \geq 2}\).

Exercise 8.2  Consider the three-state Markov chain with initial distribution \(\mu = \delta_a\) an transition probability given by the following diagram

![Diagram of a three-state Markov chain with states a, b, and c, and transition probabilities labeled as 1/2 and 1/2.

Prove that

\[
P[X_n = a] = \frac{1}{5} + \left(\frac{1}{2}\right)^n \left(\frac{4}{5} \cos \frac{n\pi}{2} - \frac{2}{5} \sin \frac{n\pi}{2}\right).
\]

Exercise 8.3  Let \(\xi_1, \xi_2, \ldots\) be i.i.d. uniform random variables on the set \(\{1, \ldots, N\}\).

(a) Show that \(X_n = |\{\xi_1, \ldots, \xi_n\}|\) is a Markov chain and compute its transition probability.

(b) Compute \(P[X_n = i]\) for \(n \geq 1\) and \(i \in \{1, \ldots, N\}\).

Submission deadline: 13:15, Apr. 18.

Location: During exercise class or in the tray outside of HG E 65.

Class assignment:

<table>
<thead>
<tr>
<th>Students</th>
<th>Time &amp; Date</th>
<th>Room</th>
<th>Assistant</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-K</td>
<td>Thu 09-10</td>
<td>HG D 7.2</td>
<td>Maximilian Nitzschnner</td>
</tr>
<tr>
<td>L-Z</td>
<td>Thu 12-13</td>
<td>HG D 7.2</td>
<td>Daniel Contreras</td>
</tr>
</tbody>
</table>
Office hours (Präsenz): Mon. and Thu., 12:00-13:00 in HG G 32.6.

Exercise sheets and further information are also available on:
http://metaphor.ethz.ch/x/2019/fs/401-3602-00L/