ETH Zürich, FS 2019
D-MATH
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Coordinator
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Applied Stochastic Processes

Exercise sheet 8

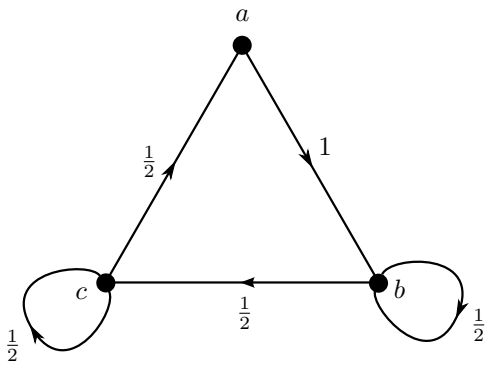
Exercise 8.1 A die is rolled repeatedly. Which of the following stochastic processes $(X_n)_{n\in\mathbb{N}}$ are Markov chains? For those that are, determine the transition probability and in (b), additionally, the n-step transition probability.

(a) Let X_n denote the number of rolls at time n since the most recent six.

(b) Let X_n denote the largest number that has come up in the first n rolls.

(c) Let X_n denote the larger number of those that came up in the rolls number $n-1$ and n (the last two rolls), and we consider $(X_n)_{n\geq 2}$.

Exercise 8.2 Consider the three-state Markov chain with initial distribution $\mu = \delta_a$ an transition probability given by the following diagram



Prove that

$$\mathbb{P}[X_n = a] = \frac{1}{5} + \left(\frac{1}{2}\right)^n \left(\frac{4}{5}\cos\frac{n\pi}{2} - \frac{2}{5}\sin\frac{n\pi}{2}\right).$$

Exercise 8.3 Let $\xi_1, \xi_2, \ldots$ be i.i.d. uniform random variables on the set $\{1, \ldots, N\}$.

(a) Show that $X_n = |\{\xi_1, \ldots, \xi_n\}|$ is a Markov chain and compute its transition probability.

(b) Compute $\mathbb{P}[X_n = i]$ for $n \geq 1$ and $i \in \{1, \ldots, N\}$.

Submission deadline: 13:15, Apr. 18.

Location: During exercise class or in the tray outside of HG E 65.

Class assignment:

Students	Time & Date	Room	Assistant
A-K	Thu 09-10	HG D 7.2	Maximilian Nitzschner
L-Z	Thu 12-13	HG D 7.2	Daniel Contreras

Office hours (Präsenz): Mon. and Thu., 12:00-13:00 in HG G 32.6.

Exercise sheets and further information are also available on:
http://metaphor.ethz.ch/x/2019/fs/401-3602-00L/