7.1. Change of variable. Define $U \subset \mathbb{R}^2$ to be the open set

$$U = \mathbb{R}^2 \cap \Big\{(x, y),\ y > 1, x > \max\Big\{y, \frac{y}{y-1}\Big\}\Big\}.$$

1. Compute the Laplace operator $\Delta = \partial_x^2 + \partial_y^2$ in the new coordinate system of U

$$\begin{cases} u = xy \\ v = x + y \end{cases} \tag{1}$$

2. Let $f : U \to \mathbb{R}, (x, y) \mapsto e^{xy} \log(xy - x - y)$. Prove f is twice differentiable and compute Δf in U.

7.2. Second order derivatives. Compute the first and the second order derivatives of the following functions.

1. $f : \mathbb{R}^2 \to \mathbb{R}, x \mapsto \cos(x)e^{-y}$.
2. Sei $n \geq 2$ and $f : \mathbb{R}^n \to \mathbb{R}, (x_1, \cdots, x_n) \mapsto \prod_{i=1}^n x_i$.

7.3. Critical points. Compute the critical points of the following functions.

1. $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x^3 + y^3 + 3xy$.
2. $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x^2 + y^2 - 2xy$.
3. $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto y(x - 1)e^{-(x^2+y^2)}$.