9.1. Differentiability The function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$f(x,y) := \begin{cases} 0 & \text{für } (x,y) = 0 \\ \frac{x^3}{\sqrt{x^2+y^2}} & \text{für } (x,y) \neq 0 \end{cases}$$

Prove that f is differentiable.

9.2. Tangential plane Given the function

$$\begin{aligned} &f\colon \Omega \to \mathbb{R} \\ &(x,y) \mapsto \sqrt{1-x^2-y^2}, \end{aligned}$$

with $\Omega = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 < 1\}$, compute the tangential plane of the graph of f at the point $(x_0, y_0, f(x_0,y_0))$.

9.3. Hessian matrix Given a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = e^{xy}(\sin(x) + 3\cos(xy))$$

Compute the Hessian matrix of f at the point $(x,y) = (0,1)$.

9.4. Line integrals Compute the following line integrals.

1. $v(x,y) = \begin{pmatrix} x^2 - 2xy \\ y^2 - 2xy \end{pmatrix}$, from $(-1,1)$ to $(1,1)$ along the curve $y = x^2$.
2. $v(x,y) = \begin{pmatrix} x^2 + y^2 \\ x^2 - y^2 \end{pmatrix}$, from $(0,0)$ to $(2,0)$ along the curve $y = 1 - |1-x|$.
3. $v(x,y,z) = \begin{pmatrix} x \\ y \\ xz - y \end{pmatrix}$, along the curve $\gamma(t) = \begin{pmatrix} t^2 \\ 2t \\ 4t^3 \end{pmatrix}$, $t \in [0,1]$.
4. $v(x,y) = \begin{pmatrix} 2a - y \\ x \end{pmatrix}$, along the curve $\gamma(t) = \begin{pmatrix} a(t - \sin(t)) \\ a(1-\cos(t)) \end{pmatrix}$, $t \in [0, 2\pi]$, with a constant $a \in \mathbb{R}$.