10.1. Conservative vector-fields

For which of the following vector-fields v does there exist a function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying $v = \nabla f$?

1. $v(x,y) = \begin{pmatrix} x-y \\ x-y \end{pmatrix}$
2. $v(x,y) = \begin{pmatrix} x^2-y \\ x^3+2xy \end{pmatrix}$
3. $v(x,y) = \begin{pmatrix} x^3+2xy \\ x^2-y \end{pmatrix}$
4. $v(x,y) = \begin{pmatrix} x^3-xy^2 \\ x^2y-y^5 \end{pmatrix}$

10.2. An example Let $V : \mathbb{R}^2 \backslash \{0\} \to \mathbb{R}^2$ be defined by

$$V(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right)$$

1. Show that V satisfies the necessary condition to be conservative in proposition 4.1.13.
2. Compute the pathintegral of V around the curve $\gamma(t) = (\sin(t), -\cos(t))$, $0 \leq t \leq 2\pi$.

10.3. Pathintegrals Compute in the following exercises the pathintegral of the vectorfield v along the path.

1. $v(x,y) = \left(\frac{x}{x^2+y^2+1}, \frac{y}{x^2+y^2+1} \right)$, along the circle $x^2+y^2-2x = 1$.
2. $v(x,y,z) = \begin{pmatrix} 2xy^2z \\ 2x^2yz \\ x^2y^2-2z \end{pmatrix}$, along the path $\gamma(t) = \left(\cos(t), \frac{\sqrt{3}}{2}\sin(t), \frac{1}{2}\sin(t) \right)$, $0 \leq t \leq 2\pi$.